Optimal monetary policy, heterogeneous expectations and consumption dispersion

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Abstract

I implement optimal monetary policy under heterogeneous expectations as in Di Bartolomeo et al. (2016) by deriving an explicit interest rate rule under timeless commitment. Implementation requires the derivation of agents' consumption decisions that incorporate the higher-order beliefs assumption of Branch and McGough (2009). As a result, "rational" agents are not sophisticated enough to have model-consistent individual consumption expectations, as assumed in Di Bartolomeo et al. (2016), even though they forecast aggregate variables correctly on average. Further, I show that the optimal interest rate rule yields substantial welfare gains compared to a rule that is derived from a conventional inflation-targeting objective as in Gasteiger (2014). The implementation of the non-optimal inflation-targeting rule already requires an increase of 14.5 percent of steady-state consumption to compensate for the higher welfare losses relative to the optimal interest rate rule when only ten percent of the population form (naive) backward-looking expectations. The presence of heterogeneous expectations requires the central bank to be extraordinarily hawkish with respect to inflation to achieve optimality. However, consumption dispersion increases with the central bank's aggressiveness towards inflation.

Keywords: Heterogeneous expectations, optimal monetary policy, consumption dispersion

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1 Introduction

The New Keynesian model emphasizes the ability of monetary policy to stabilize the macroeconomy by managing agents' expectations. Usually, optimal monetary policy is studied within a framework that assumes all agents to form their expectations rationally (Clarida et al., 1999; Woodford, 1999; McCallum, 1999). However, econometric studies based on inflation expectation surveys show that the data favors heterogeneous expectations with a certain degree of bounded rationality (Branch, 2004, 2007; Pfajfar and Santoro, 2010; Cornea-Madeira et al., 2017). For instance, Branch (2004) and Cornea-Madeira et al. (2017) find evidence for the existence of more sophisticated agents that employ a VAR-heuristic alongside agents that form (naive) backward-looking expectations. Further, Cole and Milani (2016) show that cross-equation restrictions imposed by New Keynesian models are heavily rejected under the assumption of homogeneous rational expectations when trying to match actual survey expectation data. However, these restrictions prove valuable under heterogeneous expectations, indicating that the main source of misspecification in New Keynesian models stems from misspecified (rational) expectations. Additionally, heterogeneity in forecasting schemes is confirmed by examining the expectation formation of actual human subjects in a series of laboratory experiments (Hommes, 2011; Assenza et al., 2014; Pfajfar and Žakelj, 2018). Starting from these observations, several New Keynesian models were designed that include heterogeneous expectations (Branch and McGough, 2009, 2010; De Grauwe, 2011; Massaro, 2013; Hommes and Lustenhouwer, 2019; Hagenhoff and Lustenhouwer, 2019).

What are the implications of heterogeneous expectations for monetary policy and, in particular, how should central banks set interest rates optimally given this knowledge? To answer this question I derive an optimal interest rate rule under timeless commitment[1] based on the Branch and McGough (2009) framework and a model-consistent welfare criterion following Di Bartolomeo et al. (2016). One feature of this interest rate rule is that it is extraordinarily hawkish with respect to inflation. This theoretical finding underpins the experimental finding of Assenza et al. (2014), where a central bank that is quite aggressive with respect to inflation yields more desirable aggregate outcomes in a world with heterogeneous expectations. However, the present paper additionally suggests that there may be undesirable distributional consequences as consumption dispersion increases with the aggressiveness of monetary policy towards inflation.

To describe the micro level explicitly, I derive consumption Euler equations that adequately account

[1]The timeless commitment approach assumes that the optimal commitment policy was implemented in the distant past so as to omit the first period's optimality condition that is the same as under discretion (see for instance Woodford (2003) and Woodford (2010)). The problem of the latter is that it renders the policy time-inconsistent. Hence, the drop of the initial period's optimality condition solves this problem.

for the assumption of Branch and McGough (2009) about higher-order beliefs. More specifically, the underlying model of this paper incorporates two types of agents. The more sophisticated agents, that I call "rational forecasters", are able to forecast *aggregate* variables consistent with the model predictions but are not smart enough to understand the micro level fully. In contrast, boundedly rational forecasters use a simple backward-looking heuristic instead for forecasting. Such backward-looking heuristics are broadly consistent with evidence from laboratory experiments (Hommes, 2011; Assenza et al., 2014; Pfajfar and Žakelj, 2018).

While optimal monetary policy under homogeneous rational expectations is well known and extensively studied, the strand of literature dealing with monetary policy under heterogeneous expectations is rather new. Recent advances in the literature are made by Gasteiger (2014, 2018), Di Bartolomeo et al. (2016) and Beqiraj et al. (2017). Beqiraj et al. (2017) investigate optimal discretionary monetary policy under heterogeneous expectations based on the framework developed in Massaro (2013) that includes agents that forecast over all future periods up to infinity. On the other hand, Gasteiger (2014), Gasteiger (2018) and Di Bartolomeo et al. (2016) follow the Euler-equation-learning approach of Branch and McGough (2009). Gasteiger (2014) and Gasteiger (2018) explore interest rate rules derived from an ad-hoc inflation-targeting objective while Di Bartolomeo et al. (2016) provide an extension based on a model-consistent central bank objective.

Although Di Bartolomeo et al. (2016)'s solution algorithm implies that the central bank employs expectations-based interest rate rules, they do not derive them analytically. Thus, the literature has so far not provided an optimal interest rate rule under heterogeneous expectations based on the Branch and McGough (2009) model. (Optimal) Interest rate rules are useful as they allow, in comprehensible way, to identify which variables determine the interest rate and to which degree, especially with varying degrees of heterogeneity in expectations, they should do so. In this paper, I will focus on the case where the central bank is able to commit to its policy from timeless perspective (Blake et al., 2001; Woodford, 2003) as it elegantly addresses the issue of time-inconsistency under commitment and further allows the central bank to efficiently target private sector (rational) expectations.

Additionally, I explore the role of the higher-order beliefs assumption of Branch and McGough (2009) for agents' individual consumption decisions. The authors impose this assumption as it is *necessary* for aggregation, i.e. to arrive at the same functional forms as in the canonical New Keynesian model but where the rational expectations operator is simply replaced by a weighted sum different expectation operators. Further, consumption decisions have to be made explicit as the central bank's objective function introduced by Di Bartolomeo et al. (2016) depends on consumption dispersion. This approach allows me to clarify the properties of the expectations operator, E_t^R, of "rational" forecasters in Branch and McGough (2009). It is assumed that rational forecasters, by using E_t^R, predict aggregate variables consistent with the model predictions which can, however, not be the case for expectations about the distribution of individual consumption. This is a straightforward consequence of the higher-order beliefs assumption which puts a particular (non-rational) structure on the agents believe about other agents' individual expectations. In particular, all agents believe that all other agents form the same expectations about their individual consumption as they do. Hence, even "rational forecasters" are not smart enough to sophisticatedly forecast individual consumption. The final consumption equations only depend on aggregate variables and can, therefore, be used to substitute for individual consumption in the optimal interest rate rule.

However, if an Euler equation with model-consistent individual consumption expectations as in Di Bartolomeo et al. (2016) is applied, the first-order conditions of the central bank problem under commitment can only be reduced to a second-order difference equation in one of the Lagrange-multipliers to which the solution is fairly complicated and exponentially depends on time. Consequently, an interpretable interest rate rule under commitment in this case cannot be derived. Further, it would not be possible to substitute for individual consumption so that practical implementation would require consumption of rational and boundedly rational forecasters to be observable. However, applying the consumption equation that appropriately accounts for the higher-order beliefs assumption makes the derivation of a meaningful interest rate rule under commitment possible.

Moreover, I compare the optimal interest rate rule to a micro-founded version of the policy rule in Gasteiger (2014). As already indicated, the author derives an interest rate rule under timeless commitment that recognizes the heterogeneity in expectations in the private sector equations but is optimized under a conventional ad-hoc inflation-targeting objective. The resulting interest rate rule is sub-optimal but also much simpler than the rule derived in this paper. While it is straight-forward that the optimal rule must yield lower welfare losses than the non-optimal rule derived from the inflation-targeting objective, it is not clear *by how much.*

The welfare analysis shows that the optimal interest rate rule generates substantial welfare gains, given a non-autocorrelated one-standard-deviation cost-push shock. In that case, the implementation of the non-optimal inflation-targeting rule already requires an increase of 14.5 percent of steady-state consumption to compensate for the higher welfare losses relative to the optimal interest rate rule when only ten percent of the population form (naive) backward-looking expectations. The welfare gains of the optimal interest rate rule crucially depend on the relative fraction of agent types. The optimal interest rate rule performs relatively better the higher the fraction of boundedly rational forecasters.

Finally, I find that consumption dispersion is not necessarily lower under the optimal rule compared the non-optimal inflation-targeting rule, even though the former explicitly incorporates consumption heterogeneity as opposed to the latter. This is because consumption dispersion increases with the central bank's aggressiveness towards inflation, as rational and boundedly rational forecasters' consumption decisions become more unequal for stronger reactions of the policy rate. As a consequence, there is also a (local) trade-off between minimizing welfare losses and reducing consumption dispersion.

The remainder of the paper is organized as follows. The underlying model including the modified consumption rules are presented in Section 2. The optimal interest rate rule under heterogeneous expectations is derived in the subsequent section. Section 4 shows the impulse responses under optimal monetary policy with an emphasis on the different consumption paths of rational and boundedly rational forecasters, followed by the welfare analysis in Section 5. Finally, the conclusion is given in Section 6.

2 Model

In this section, I introduce the underlying model and derive individual consumption decisions that incorporate the higher-order beliefs assumption of Branch and McGough (2009). The economy is assumed to be populated by an exogenous fraction α of rational forecasters (R) which have rational (model-consistent) expectations with respect to *aggregate* variables and a fraction $1-\alpha$ of boundedly rational forecasters (B) that employ a simple backward-looking heuristic. The general forecasting rule of boundedly rational forecasters takes the form of $E_t^B x_{t+1} = \theta^2 x_{t-1}$ for some variable x while rational forecasters simply use the expected value, $E_t^R = E_t$, for forecasting aggregate variables. Backward-looking expectations for $\theta < 1$ are called steady-state-reverting, for $\theta = 1$ naive and for $\theta > 1$ trend-setting. Steady-state-reverting expectations constitute a stabilizing force while trend-setting expectations imply a further amplification of macroeconomic variables.

Assuming perfect consumption insurance *within* each of the two agent groups the model can be expressed in terms of two representative agents (RA). This approach, therefore, abstracts from further individual characteristics in order to isolate the effects of heterogeneous expectations on individual consumption (dispersion) and aggregate variables, as well as their implications for monetary policy. Both RA's aim at maximizing their individual expected discounted lifetime utility $E_0^\tau \sum_{t=0}^{\infty} \beta^t U_t$ given their subjective expectations E_0^τ with $\tau \in \{R, B\}$. However, as in Branch and McGough (2009) agents follow Euler-equation learning, i.e. they disregard their intertemporal budget constraint as an optimality condition and solely base their consumption decision on the variational intuition of the consumption Euler equation.

The period utility function is of CES-form and is given by

$$U_t = \frac{(C_t^\tau)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{(Y_t^\tau)^{1+\eta}}{1+\eta} \tag{1}$$

with C_t^τ being consumption of the RA of type τ, Y_t^τ the output that each type-τ RA produces, $\frac{1}{\sigma}$ the coefficient of relative risk aversion and η the elasticity of marginal disutility of producing output. Agents must satisfy their real budget constraint

$$C_t^\tau + B_t^\tau = \frac{1+i_{t-1}}{\Pi_t} B_{t-1}^\tau + \Psi_t^\tau \tag{2}$$

with B_t^τ being real bond holdings, i_{t-1} the nominal interest rate in $t-1$, Π_t gross inflation and Ψ_t^τ real income of agent τ.

All agents in this economy are assumed to believe that all other agents will form the same expectations as they do. Branch and McGough (2009) explicitly emphasize that this assumption is *necessary* for aggregation. I explicitly show below that this assumption implies that "rational" forecasters are not fully rational in the conventional sense and therefore cannot have model-consistent individual consumption expectations. Assuming rational forecasters to possess rational individual consumption expectation, as in Di Bartolomeo et al. (2016), *implies too much rationality* to be consistent with the *necessary* higher-order beliefs assumption. Incorporating the higher-order beliefs assumption into the consumption decisions of agents is further crucial as an interpretable interest rate rule under timeless commitment cannot be derived otherwise (see Appendix C).

I will show later on that the central bank's welfare criterion can be re-written using market clearing in a way that it only depends on the consumption decision of rational forecasters. Hence, for now I focus on the consumption Euler equation for $\tau = R$ which is given by

$$(C_t^R)^{-\frac{1}{\sigma}} = \beta E_t^R \left[(C_{t+1}^R)^{-\frac{1}{\sigma}} \frac{1+i_t}{\Pi_{t+1}} \right]. \tag{3}$$

Log-linearizing (3) gives

$$c_t^R = E_t^R c_{t+1}^R - \sigma(i_t - E_t^R \pi_{t+1}) \tag{4}$$

where lower case letters indicate log-deviations from the zero-inflation steady state. Forward iteration yields

$$c_t^R = E_t^R c_\infty^R - \sigma \sum_{k=0}^{\infty} (i_{t+k} - E_t^R \pi_{t+k+1}). \tag{5}$$

Rational forecasters assume that the Euler equation of boundedly rational forecasters will also be satisfied and that market clearing $y_t = \alpha c_t^R + (1-\alpha) c_t^B$ holds. Writing market clearing one period forward and inserting equation (5), and equivalently the forward-iterated consumption Euler equation for boundedly rational forecasters, yields

$$\begin{aligned} E_t^R y_{t+1} = E_t^R \Big[&\alpha (E_t^R c_\infty^R - \sigma E_{t+1}^R \sum_{k=1}^{\infty} (i_{t+k} - \pi_{t+k+1})) \\ &+ (1-\alpha)(E_t^B c_\infty^B - \sigma E_{t+1}^B \sum_{k=1}^{\infty} (i_{t+k} - \pi_{t+k+1})) \Big]. \end{aligned} \tag{6}$$

Note that (6) contains higher-order beliefs, i.e. beliefs of rational forecasters E_t^R about the beliefs of boundedly rational forecasters E_t^B and E_{t+1}^B. In order to arrive at the IS curve that has the same functional form as in the model under homogeneous rational expectations, Branch and McGough (2009) impose a specific (non-rational) structure on higher-order beliefs. In the context of the present paper, the assumption states that agents' believe that all other agents will forecast their individual consumption in the same way they do. Mathematically and in the case of rational forecasters: $E_t^R E_{t+k}^B c_{t+l} = E_t^R c_{t+l}$ with $l > k$. Hence, boundedly rational expectations just drop out under this assumption. Further, note that making an alternative assumption, e.g. allowing rational forecasters to be fully rational, would result in a different system of aggregate equations (see Hagenhoff and Lustenhouwer, 2019).

Using the higher-order beliefs assumption and the law of iterated expectations at the individual level yields

$$E_t^R y_{t+1} = E_t^R y_\infty - \sigma \sum_{k=1}^{\infty} (i_{t+k} - E_t^R \pi_{t+k+1}). \tag{7}$$

It becomes obvious that (7) cannot hold under conventional rational expectations, i.e. when $E_t^R = E_t$ would hold, as boundedly rational expectations, E_t^B and E_{t+1}^B, would *not drop out* and thus show up in (7). Mathematically written: $E_t E_t^B = E_t^B$ and $E_t E_{t+1}^B = E_{t+1}^B$, which would contradict the higher-order beliefs assumption of Branch and McGough (2009). Equation (7) would only hold under conventional rational expectations when boundedly rational forecasters were absent, i.e. under *homogeneous rational expectations*. In this case (6) would collapse to (7) without any further assumption.

The main crucial point that follows from this consideration is that rational forecasters in this model are not smart enough to have model-consistent expectations with respect to the distribution of individual consumption *when there is heterogeneity.* Thus, assuming model-consistent individual consumption expectations in the Euler equation of rational forecasters, as in Di Bartolomeo et al. (2016), is inconsistent with the underlying framework.

Using (7) to replace the infinite sum in (5), one obtains

$$c_t^R = E_t^R y_{t+1} + E_t^R (c_\infty^R - y_\infty) - \sigma(i_t - E_t^R \pi_{t+1}) \tag{8}$$

which is the true consumption decision of rational forecasters satisfying the higher-order beliefs assumption from above.

Equation (8) could have been derived for the general case of agent τ as the assumption on higher-order beliefs holds for both agent types. In the general case (8) reads

$$c_t^\tau = E_t^\tau y_{t+1} + E_t^\tau (c_\infty^\tau - y_\infty) - \sigma(i_t - E_t^\tau \pi_{t+1}). \tag{9}$$

To be consistent with Branch and McGough (2009), I assume that agents believe to be back in steady state in the long-run.[2] In this case $E_t^\tau(c_\infty^\tau - y_\infty) = 0$ holds and, thus, (9) becomes

$$c_t^\tau = E_t^\tau y_{t+1} - \sigma(i_t - E_t^\tau \pi_{t+1}). \tag{10}$$

From (10) one can infer that agents only forecast aggregate variables when making consumption decisions. Note that, as rational forecasters have rational expectations with respect to aggregate variables, E_t^R can be replaced by E_t in the consumption decision of rational forecasters.

Using goods market clearing and (10) the IS curve is given by

$$y_t = \alpha E_t y_{t+1} + (1-\alpha)\theta^2 y_{t-1} - \sigma(i_t - \alpha E_t \pi_{t+1} - (1-\alpha)\theta^2 \pi_{t-1}). \tag{11}$$

Further, output is produced under monopolistic competition. Calvo pricing is assumed where a fixed fraction ξ_p of yeoman farmers cannot reset their prices in a given period (Calvo, 1983). Price dispersion arises because, first, optimal prices are different *between* expectation types since they depend on *expected* future marginal costs and, second, they differ *within* each type due to the fact that only a fraction of

[2]Branch and McGough (2009) assume that agents agree on expected differences in expected limiting consumption, so that long-run expectations drop out when aggregating the individual consumption decisions. As the authors are interested only in aggregate dynamics, there is no need to explicitly specify what these expectations actually are. However, this model differs because optimal monetary policy depends on consumption dispersion. Branch and McGough (2009, p.1041) mention that one assumption, consistent with their own, would be to assume that agents believe to be back in the steady state in the long-run which I employ here.

firms can reset prices. Thus, the Phillips-curve can be derived as

$$\pi_t = \alpha\beta E_t\pi_{t+1} + (1-\alpha)\beta\theta^2\pi_{t-1} + \kappa y_t + e_t \tag{12}$$

with $\kappa = \frac{(1-\xi_p)(1-\beta\xi_p)(\eta+\sigma^{-1})}{\xi_p(1+\epsilon\eta)}$ where ϵ is the price elasticity of demand for a differentiated good.[3] As in Di Bartolomeo et al. (2016) the Phillips curve is augmented with a random cost-push shock e_t.[4]

Note that inflation and output exhibit some degree of *persistence* due to the presence of backward-looking expectations. The degree of persistence depends on the fraction of boundedly rational forecasters $1-\alpha$ and their forecasting coefficient θ. The higher the two parameters the higher the degree of persistence. This persistence is also recognized by rational forecasters which feeds back into the aggregate equations (11) and (12) via rational expectations. Hence, the interaction between rational and boundedly rational expectations leads, ceteris paribus, to an *amplification* of shocks as identified by Gasteiger (2018). However, the central bank's ability to manipulate (rational) expectations by setting interest rates accordingly is a powerful tool to counteract this amplification effect.

The model is calibrated as in Di Bartolomeo et al. (2016) for the US economy following Rotemberg and Woodford (1997) with the time unit being one quarter.

$\alpha = 0.7$	$\theta = 1$	$\beta = 0.99$	$\sigma = 6.25$	$\epsilon = 7.84$	$\eta = 0.47$	$\xi_p = 0.66$

Table 1: Baseline calibration

3 An optimal interest rate rule from a timeless perspective

In this section I derive, first, an optimal timeless interest rate rule from a model-consistent welfare criterion and, second, a non-optimal rule under a conventional inflation-targeting objective as in Gasteiger (2014).

3.1 Loss functions

As in Gasteiger (2014, 2018) and Di Bartolomeo et al. (2016) the central bank aims at maximizing social welfare. I follow the approach of Di Bartolomeo et al. (2016) where the central bank exploits its detailed knowledge about the heterogeneity in expectations and minimizes a social welfare loss that is a second-order approximation of household utility (1). The intertemporal second-order approximated aggregate

[3] For the derivation of the Phillips curve (12) please refer to Branch and McGough (2009).

[4] This shock can, for instance, be micro-founded by assuming an exogenous time-varying wage mark-up as in Galí (2015).

welfare loss can be derived as

$$\mathcal{W} = -\frac{\bar{C}U_C}{2}\sum_{t=0}^{\infty}\beta^t L_t + t.i.p. \tag{13}$$

with

$$L_t = \left(\eta + \frac{1}{\sigma}\right) y_t^2 + (\epsilon^2\eta) var_i(p_t(i)) + \frac{1}{\sigma} var_i(c_t(i)). \tag{14}$$

and

$$var_i(p_t(i)) = \delta\pi_t^2 + \frac{\delta\xi_p(1-\alpha)}{\alpha}\left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa\frac{c_t^B + \eta\sigma y_t}{1+\eta\sigma}\right]^2 \tag{15}$$

$$var_i(c_t(i)) = \alpha(1-\alpha)(c_t^R - c_t^B)^2. \tag{16}$$

where $\delta = \frac{\xi_p}{(1-\beta\xi_p)(1-\xi_p)}$ is a measure of price stickiness and $t.i.p.$ are the terms independent of policy.

Using market clearing to eliminate c_t^B as well as (15) and (16), loss function (14) can be rewritten as

$$\begin{aligned} L_t =& \Gamma_1 y_t^2 + \Gamma_2\pi_t^2 + \Gamma_3\pi_{t-1}^2 + \Gamma_4(c_t^R)^2 \\ &+ \Gamma_5 y_t c_t^R + \Gamma_6\pi_t c_t^R + \Gamma_7\pi_{t-1}c_t^R + \Gamma_8\pi_t\pi_{t-1} + \Gamma_9\pi_t y_t + \Gamma_{10}\pi_{t-1}y_t \end{aligned} \tag{17}$$

where the Γ_x-coefficients are given in the Appendix B.1.

Under homogeneous rational expectations, i.e for $\alpha = 1$, price dispersion reduces to $var_i(p_t(i)) = \delta\pi_t^2$ and $var_i(c_t(i))$ to zero. Hence, in this case (14) reduces to

$$L_t^{\alpha=1} = \left(\eta + \frac{1}{\sigma}\right) y_t^2 + \epsilon^2\eta\delta\pi_t^2. \tag{18}$$

Equation (14) can be called the model-consistent loss function, i.e. it is consistently microfounded under heterogeneous expectations as assumed in this paper, and (18) the conventional[5] inflation-targeting loss function.

In general agents dislike volatility due to the concave nature of their utility function. However, the weight that is placed on inflation in second-order approximated utility functions in the canonical New Keynesian model such as (18) is usually considerably higher compared to the weight on output (see Woodford, 2003 or Galí, 2015). This reflects that price dispersion, due to inflation and sticky prices, is *the* source of inefficiency in the baseline model, quickly resulting in relatively large welfare losses. In case of the conventional inflation-targeting loss (18), the weight on inflation is roughly 160 times the weight

[5] It is conventional in the sense that it can be derived from the conventional three-equation NK model under homogeneous rational expectations.

on output under baseline calibration.[6]

However, when producers have heterogeneous expectations, price dispersion arises not only due to sticky prices but also because they have different expectations regarding future inflation and marginal costs, as reflected by (15). The weight on contemporaneous inflation in the model-consistent loss function (17) is roughly 230 times of the weight on contemporaneous output under baseline calibration with a 70 percent of rational forecasters, and increases to 270 times of the weight on contemporaneous output for 50 percent of rational forecasters. Additionally, the weights on lagged inflation and on the interaction between contemporaneous and lagged inflation are non-negligible. Hence, inflation results in even higher welfare losses through the price dispersion channel under heterogeneous expectations. However, even though inflation explains most of the results in the the welfare analysis in Section 5, there is still a trade-off between inflation and output (and consumption dispersion) under a cost-push shock that can be important in some cases.

Further, an interesting novelty of (14) is the appearance of consumption dispersion, i.e. the cross-sectional variance in consumption $var_i(c_t(i))$. It should be noted, however, that the weight on consumption dispersion in (14) is even smaller compared to the weight on output. This indicates that agents might accept a certain degree of heterogeneity when the economy is relatively stable instead. This finding is also in line with Debortoli and Galí (2017) who derive a model-consistent loss function in a Two Agent New Keynesian (TANK) model and show that it depends on a measure of heterogeneity but where the corresponding weight relative to inflation and output is also very low.

3.2 An optimal interest rate rule under timeless commitment

The central bank is assumed to set its interest rate so as to minimize the model-consistent loss function (17) or, respectively, the conventional inflation-targeting objective (18) subject to the private sector equations

$$y_t = \alpha E_t y_{t+1} + (1-\alpha)\theta^2 y_{t-1} - \sigma[i_t - \alpha E_t \pi_{t+1} - (1-\alpha)\theta^2 \pi_{t-1}] \tag{19}$$

$$\pi_t = \alpha\beta E_t \pi_{t+1} + (1-\alpha)\beta\theta^2 \pi_{t-1} + \kappa y_t + e_t \tag{20}$$

$$c_t^R = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}). \tag{21}$$

Minimizing the conventional inflation-targeting objective (18) subject to the Phillips curve (20) under

[6] The fact that the weight on inflation is high relative to the weight on output is general and robust with respect to the calibration.

timeless commitment gives the inflation-targeting interest rate rule

$$i_t = \gamma_1 y_{t-1} + \gamma_2 E_t y_{t+1} + \gamma_3 \pi_{t-1} + \gamma_4 E_t \pi_{t+1} + \gamma_5 e_t \tag{22}$$

with

$$\gamma_1 = \frac{(1-\alpha)\theta - \alpha}{\sigma} + \alpha \frac{\delta \epsilon^2 \eta \kappa^2}{1 + \eta\sigma + \delta\epsilon^2\eta\kappa^2\sigma} \tag{23}$$

$$\gamma_2 = \alpha \frac{1}{\sigma} - \frac{(1-\alpha)}{\sigma} \left[\frac{\beta^2\theta^2(1+\eta\sigma)}{1 + \eta\sigma + \delta\epsilon^2\eta\kappa^2\sigma} \right] \tag{24}$$

$$\gamma_3 = (1-\alpha) \left[\frac{\theta^2(1 + \eta(\sigma + \delta\epsilon^2\kappa(\beta + \kappa\sigma)))}{1 + \eta\sigma + \delta\epsilon^2\eta\kappa^2\sigma} \right] \tag{25}$$

$$\gamma_4 = \alpha \left[1 + \frac{\beta\delta\epsilon^2\eta\kappa}{1 + \eta\sigma + \delta\epsilon^2\eta\kappa^2\sigma} \right] \tag{26}$$

$$\gamma_5 = \frac{\delta\epsilon^2\eta\kappa}{1 + \eta\sigma + \delta\epsilon^2\eta\kappa^2\sigma}. \tag{27}$$

Equation (22) is similar to the rule derived by Gasteiger (2018) and Gasteiger (2014).[7] Note that timeless commitment introduces persistence even in the absence of boundedly rational forecasters, i.e. γ_1 reduces to $\frac{\delta\epsilon^2\eta\kappa^2}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma} - \frac{1}{\sigma}$ and does not vanish for for $\alpha = 1$.

The *optimal* commitment interest rate rule can be obtained by minimizing (17) subject to the private sector equation (19), (20) and (21) under timeless commitment as

$$\begin{aligned} i_t =& \Omega_1 E_t \pi_{t+1} + \Omega_2 E_t \pi_{t+2} + \Omega_3 \pi_{t-3} + \Omega_4 \pi_{t-2} + \Omega_5 \pi_{t-1} + \Omega_6 E_t y_{t+1} + \Omega_7 E_t y_{t+2} \\ &+ \Omega_8 y_{t-2} + \Omega_9 y_{t-1} + \Omega_{10} E_t c^R_{t+1} + \Omega_{11} E_t c^R_{t+2} + \Omega_{12} c^R_{t-2} + \Omega_{13} c^R_{t-1} + \Omega_{14} e_t \end{aligned} \tag{28}$$

where the reaction coefficients Ω_x and derivations are given in the Appendix B.2. A first inspection of (28) shows that the central bank reacts to output and inflation as usual but also to individual consumption of rational forecasters due to the consumption dispersion dimension. However, a more striking observation is that the central bank finds it optimal to react to lags and leads of all variables ranging from $t-2$ to $t+2$ (and additionally $t-3$ for inflation). This will be clarified further below. Note that under homogeneous rational expectations, $\alpha = 1$, all coefficients associated with heterogeneous expectations vanish as well as the additional coefficients due to commitment, except for y_{t-1} which can be seen in table (5) in the Appendix B.4.

Further, efficiently controlling private sector rational expectations requires the central bank to induce

[7] The author uses a non-micro-founded version of (18), i.e. $L_t = \frac{1}{2}(\pi_t^2 + \omega y_t^2)$. Setting $\omega = \frac{\sigma\eta+1}{\epsilon^2\eta\delta\sigma}$ and calculating through the optimization problem yields the interest rate rule (22).

substantial persistence by including terms with timing $t-2$ and $t-3$ in the optimal rule (28). This becomes clear when one would construct a hypothetical case in which the central bank observes and reacts to expectations but neglects the endogenous nature of rational expectations with respect to its policy.[8] In such a hypothetical case, the interest rate rule would be given by[9]

$$\begin{aligned} i_t =& \Omega_1^* y_{t-1} + \Omega_2^* E_t y_{t+1} + \Omega_3^* E_t y_{t+2} + \Omega_4^* \pi_{t-1} + \Omega_5^* E_t \pi_{t+1} + \Omega_6^* E_t \pi_{t+2} \\ &+ \Omega_7^* E_t c_{t+1}^R + \Omega_8^* E_t c_{t+2}^R + \Omega_9^* e_t \end{aligned} \tag{29}$$

where all terms with timing $t-2$ and $t-3$ drop out.

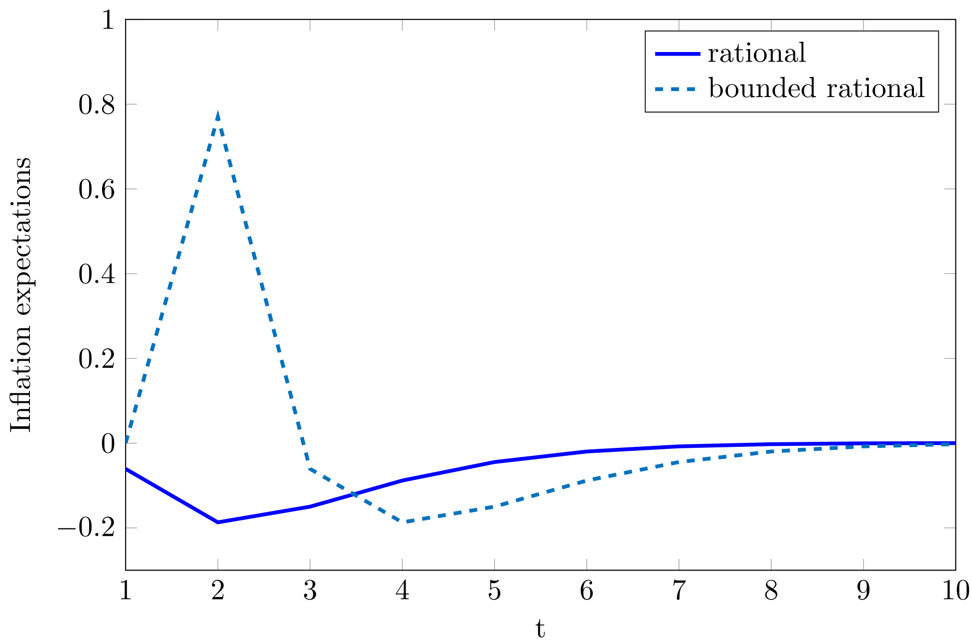

Figure 1: Individual inflation expectations in percentage deviation from steady state following a single, non-autocorrelated cost-push shock of one percent with monetary policy given by (28).

Moreover, to understand the appearance of the $t+2$ terms consider Figure (1) which displays the reaction of the inflation expectations of both agent types following a one standard deviation i.i.d. cost-push shock under the policy rule (28). Since all subsequent shock realizations are zero and rational forecasters know the true aggregate equations, they have de facto perfect foresight. Thus, rational forecasters' expectations in $t=1$ about inflation in $t+1$ will be correct, i.e. $E_t\pi_{t+1} = \pi_{t+1}$. However, the backward-looking expectations of boundedly rational forecasters in t about inflation in $t+1$ will be zero, i.e. $E_t^B \pi_{t+1} = \pi_{t-1} = 0$ (where $\theta = 1$ for simplicity). In period $t+1$ boundedly rational forecasters will

[8] Such a behavior would, for instance, be consistent with a boundedly rational central bank that operates under some sort of "limited" commitment. However, I use this only for the sake of exposition.

[9] the reaction coefficients are given in the Appendix B.2

expect inflation to *increase* drastically in $t+2$ as their expectations are based on the period where the shock hits, i.e. $E^B_{t+1}\pi_{t+2} = \pi_t$. On the contrary, rational forecasters correctly expect inflation to *decrease* further as the central bank increases the nominal interest rate a second time (see impulse responses in Figure 2 in section 4). Thus, the central bank induces the different expectations in $t+1$ about $t+2$ to diverge transitorily. Consequently, rational expectations counteract boundedly rational ones which stabilizes inflation. However, as price dispersion is additionally caused by *differences* in expectations about inflation and marginal costs, the central bank includes terms with $t+2$ timing into its interest rate rule to prevent sub-optimally high dispersion in expectations.

Moreover, it seems, at first glance, that for practical implementation the optimal interest rate rule (28) requires to observe individual consumption of rational forecasters which is, however, not observable in reality. As already indicated, an advantage of the consumption decision (10) is that it only depends on aggregate variables as a result of the explicit incorporation of the higher-order beliefs assumption. Therefore, it is possible to substitute for individual consumption so that the optimal interest rate rule is merely a function of several leads and lags of aggregate variables.

I do not discuss determinacy issues as the model is determinate for all considered parameter constellations. There are two reasons for this. First, an expectations-based interest rate rule is derived, i.e. it properly accounts for private sector expectations which are known to perform exceptionally well as opposed to fundamentals-based reaction functions (Evans and Honkapohja, 2006). Second, the interest rate rule is derived from the model-consistent loss function and is, therefore, optimal.[10]

4 Impulse Responses

This section briefly describes the simulation outcomes under baseline calibration given in Table 1. The impulse responses of a one percent i.i.d cost-push shock with monetary policy given by (28) are depicted in Figure 2. The aggregate behavior of the model is straightforward: taking into account subjective expectations, the real interest rates of both agent types, $r^\tau_t = i_t - E^\tau_t \pi_{t+1}$, increase due to an increase of the nominal rate by the central bank. Hence, both agent types cut their individual consumption which leads to a quite severe recession which counteracts the cost-push shock to some extent. Consequently, inflation increases by less than one percent. Thus, the central bank finds it optimal to be extraordinarily hawkish with respect to inflation which comes at the cost of a significant recession.

On the individual level, the disparity between the consumption adjustment paths of both agent types becomes obvious. While boundedly rational forecasters cut their consumption by approximately three

[10]Given the restrictions imposed by the timeless commitment assumption.

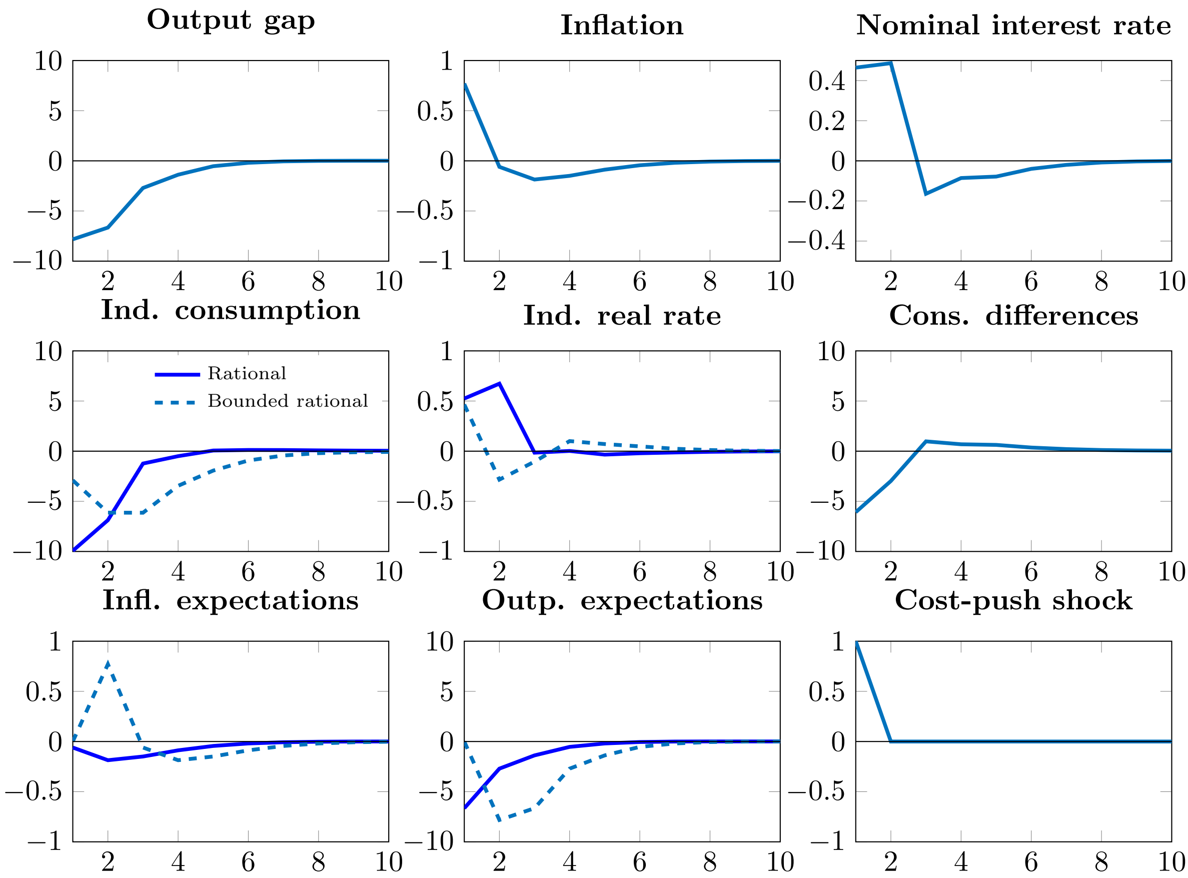

Figure 2: Impulse responses in %-deviations from steady state following a single, non-autocorrelated cost-push shock.

percent on impact, rational forecasters decrease consumption by almost ten percent. This is, first, because of substantially negative rational output gap expectations and, second, due to a slightly higher subjective real interest rate. On the other hand, as boundedly rational forecasters are backward-looking, their output gap expectations are zero on impact and, therefore, cut their consumption because of the increase in the subjective real interest rate only. This results in a consumption cut that is far smaller compared to rational forecasters and thereby in significant differences in individual consumption on impact.[11]

Thus, boundedly rational forecasters seem to be better off than rational ones at first. However, boundedly rational forecasters make less smart decisions than rational forecasters *by definition*. This becomes clear when looking at the following periods where boundedly rational forecasters have to pay for their initially higher consumption by giving up a lot of future consumption. Specifically, one can observe that boundedly rational output gap expectations in the second period $(t+1)$ *drop* drastically to $E^B_{t+1}y_{t+2} = y_t$ which is approximately minus 8 percent, resulting in a further cut of consumption. This is the case even though the subjective real interest rate of boundedly rational forecasters becomes negative

[11] In this context, I simply define a weighted difference between consumption of rational and boundedly rational agents as $\alpha c_t^R - (1-\alpha)c_t^B$.

which is due to the jump of their inflation expectations to $E^B_{t+1}\pi_{t+2} = \pi_t$. At the same time, output gap expectations of rational forecasters *increase* as the output gap recovers. From this period onwards rational forecasters are able to consume more than boundedly rational ones for a prolonged time.

5 Welfare evaluation

This section provides a comparison between the optimal interest rate rule (28) and the non-optimal inflation-targeting rule (22) in terms of welfare and a brief discussion on consumption dispersion. In particular, I analyze the welfare consequences of these rules following a one-percent i.i.d. cost-push shock as before. It should be noted that shocks to inflation directly (and hence to price dispersion) induce high welfare losses. The reason is, first, that price dispersion leads to dispersion in imperfectly substitutable individual production and, therefore, to losses in the final consumption bundle and, second, that output itself needs to be contracted in order to bring down inflation. Further, I will restrict the analysis in this Section to the case of naive expectations, i.e. $\theta = 1$, of boundedly rational forecasters for simplicity.

5.1 Optimal vs. inflation-targeting rule

In the following, I show *to what extent* the optimal interest rate rule (28) yields lower welfare losses compared to rule (22) depending on the fraction of rational forecasters. To that end, I compute consumption equivalent welfare losses following Ravenna and Walsh (2011). Let

$$\mathcal{W}^O = -\frac{\bar{C}U_c}{2}E_t\sum_{t=0}^{\infty}\beta^t L^O_t + t.i.p. = -\frac{\bar{C}U_c}{2(1-\beta)}L^O + t.i.p. \tag{30}$$

be the welfare loss under the optimal commitment policy (28), and

$$\mathcal{W}^{IT} = -\frac{\bar{C}U_c}{2}E_t\sum_{t=0}^{\infty}\beta^t L^{IT}_t + t.i.p. = -\frac{\bar{C}U_c}{2(1-\beta)}L^{IT} + t.i.p. \tag{31}$$

be the welfare loss under the non-optimal inflation-targeting objective (22), where instantaneous losses are measured by (17) in both cases. The welfare loss of implementing policy (22) instead of (28) can be measured as the percentage increase of steady state consumption, CE, satisfying

$$\frac{U((1+CE)\times\bar{C})}{1-\beta} + \mathcal{W}^{IT} = \frac{U(\bar{C})}{1-\beta} + \mathcal{W}^O. \tag{32}$$

Inserting consumption utility $U(\bar{C}) = \frac{\bar{C}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$,(30), (31) and $U_c = \bar{C}^{-\frac{1}{\sigma}}$, and solving for CE gives[12]

$$CE = \left(1 - \frac{\sigma - 1}{2\sigma}\left(L^O - L^{IT}\right)\right)^{\frac{\sigma}{\sigma-1}} - 1. \tag{33}$$

Table 2 shows absolute losses L^O and L^{IT} (second and third column) as well as the consumption equivalent welfare costs, CE, (fifth column) for different fractions of rational agents, α. The other columns of Table 2 are discussed further below.

α	L^{T*}	L^O	L^{IT}	L^{IT*}	CE	CE^*
0.95	161.65	127.077	127.138	127.08	0.030	0.006
0.9	183.82	144.988	145.274	144.99	0.145	0.034
0.7	277.56	243.455	246.632	244.02	1.744	0.286
0.5	400.36	394.547	402.428	395.45	4.693	0.47

Table 2: Column 2-4 show absolute welfare losses for *numerically optimized* simple Taylor rule (T^*), the optimal (O) interest rate rule (28), the inflation-targeting (IT) rule (22) and the inflation-targeting rule where the all reaction coefficients are *numerically optimized* (IT^*). Column 5-6 depict consumption equivalent welfare costs of implementing the non-optimal inflation-targeting (IT) interest rate rule (22) relative to the optimal (O) commitment policy (28) in percent, CE, and also for the *numerically optimized* inflation-targeting (IT^*) rule relative to the optimal rule, CE^*. All shown values are calculated for different fractions of rational agents, α.

A first, more trivial, observation is that absolute welfare losses increase with the fraction of boundedly rational forecasters for both interest rate rules. The higher the fraction of naive forecasters, the more persistent the deviations of variables from steady state and, therefore, the higher the long-run variances. Further, welfare losses are naturally lowest under the optimal interest rate rule (28). Deriving the interest rate rule from the conventional inflation-targeting (IT) objective (18) is costly in terms of consumption equivalents, CE. The implementation of the non-optimal inflation-targeting rule (22) already requires an increase of 14.5 percent of steady-state consumption to compensate for the higher welfare losses relative to the optimal interest rate rule (28) when only ten percent of the population form (naive) backward-looking expectations. Welfare costs become substantially higher for higher fractions of boundedly rational forecasters (lower α). Hence, the optimal interest rate rule (28) yields considerable welfare gains when the underlying economy features high degree of bounded rationality.

The difference between the two rules can be explained by their relative ability to stabilize inflation.

[12]Note that the terms independent of policy (*t.i.p.*) are the same for both $\mathcal{W}^{IT}$ and $\mathcal{W}^O$ and, therefore, drop out.

The first two rows in Table 3 depict the variances of inflation and output for different values of rational forecasters, α, for both interest rate rules. In general, the optimal interest rate rule (first row) yields lower inflation but higher output volatility across all fractions of rational forecasters, α. This is a straightforward implication of the relatively higher weights on inflation in the model-consistent loss function (17) compared to the conventional inflation-targeting objective (18), as discussed in Section 3.1. While the differences in output and inflation volatility are relatively low for $\alpha = 0.95$, they become substantial for higher fractions of boundedly rational forecasters, resulting in quite extreme consumption equivalents.

Further, as discussed in Section 3.2, the optimal interest rate rule (28) depends on much more leads and lags of all variables compared to the non-optimal inflation-targeting rule (22). This raises the question whether it is the absence of these leads and lags that makes (22) sub-optimal or if it is rather an issue of weighting the different variables in the interest rate rule, or both. To answer this question, I *numerically optimize* the reaction coefficients in (22) with respect to welfare (17). The corresponding absolute welfare losses, L^{IT*}, and consumption equivalent welfare costs relative to the optimal commitment policy, CE^{*}, can be found in columns four and six in Table 2, respectively. Consumption-equivalent welfare costs, CE^{*}, *substantially* decrease relative to the consumption-equivalent welfare costs, CE. This indicates that the inflation-targeting interest rate rule (22) is sub-optimal especially because of the sub-optimal weighting of its different terms.[13] However, for $\alpha = 0.7$ and $\alpha = 0.5$ the welfare gains of the optimal interest rate rule (28) are still non-negligible, suggesting that its additional leads and lags are, at least to some extent, important when heterogeneity increases.

Finally, for the sake of comparison, I add a numerically optimized simple Taylor rule with contemporaneous inflation and output to the analysis. Absolute welfare losses, L^{T*}, are shown in the first column of Table 2 and the corresponding variances of inflation and output can be found in the last row of Table 3. For a fraction of 70 percent of rational forecasters, or higher, the numerically optimized simple Taylor rule is the worst performing among all alternatives. However, interestingly, for 50 percent of rational forecasters it trumps the analytically derived inflation-targeting rule (22). Note that the Taylor rule is numerically optimized under the model-consistent loss function (17) which, in particular, recognizes the complex nature of price dispersion which is highest around $\alpha = 0.5$. On the other hand, the inflation-targeting rule is derived from the conventional inflation-targeting objective (18) which abstracts from

[13] I started by optimizing the coefficients on inflation (expectations) and the cost-push shock while "fixing" the coefficients on output to the analytically derived ones, (23) and (24). The resulting absolute welfare losses and consumption equivalents are nearly the same as in Table 2, indicating that the sub-optimal weighting in the analytically derived inflation-targeting rule (22) is mainly because of *sub-optimal weighting of inflation expectations and the shock.*

rule	$Var(y)$				$Var(\pi)$			
	$\alpha = 0.95$	$\alpha = 0.9$	$\alpha = 0.7$	$\alpha = 0.5$	$\alpha = 0.95$	$\alpha = 0.9$	$\alpha = 0.7$	$\alpha = 0.5$
Optimal	41.23	50.66	115.17	231.69	0.58	0.60	0.66	0.67
IT	38.62	44.80	88.98	179.89	0.59	0.62	0.75	0.83
IT^*	41.37	50.93	116.82	232.55	0.58	0.60	0.66	0.66
T^*	42.00	59.98	149.45	240.37	0.78	0.78	0.70	0.65

Table 3: Theoretical variances of inflation and output under the optimal (O) rule (28), the inflation-targeting (IT) objective (22), the *numerically optimized* inflation-targeting (IT^*) rule and the *numerically optimized* simple Taylor rule (T^*) for different values of rational forecasters α.

the complex nature of price dispersion.[14] Further, in the case of $\alpha = 0.5$, the numerically optimized Taylor rule generates lower inflation volatility than the optimal commitment policy. This is, however, not welfare maximizing as the Taylor rule implies the highest output volatility among all interest rate rules and for all α. This indicates that, although inflation is the most important driver of welfare, there is still a welfare-relevant trade-off between inflation and output.

5.2 Inflation and welfare vs. consumption dispersion

In this section, I briefly discuss the issue of distribution, measured by the cross-sectional variance in consumption (16), and monetary policy. It should be noted, however, that *welfare losses* due to consumption dispersion are in general very low under the model-consistent loss function (17), where individual utilities are weighted equally and simply summed up. Of course, different conclusions may be reached when a social planer would attach higher weights to consumption dispersion.

Table 4 shows consumption dispersion (CD) and the variance of inflation for both the optimal interest rate rule (O) and inflation-targeting rule (IT) for different fractions of rational forecasters, α. Interestingly, consumption dispersion is *higher* under the optimal interest rate rule (28) compared to the (analytically derived) inflation-targeting rule (22) for all α. This is the case even though the former explicitly incorporates consumption heterogeneity as opposed to the latter. At the same time, the optimal interest rate rule yields lower inflation volatility, as discussed in the previous section. This indicates that the model additionally implies a trade-off between stabilizing inflation and consumption heterogeneity.

[14]For $\alpha \neq 0.5$ (where the case of $\alpha < 0.5$ is not explicitly shown here) the numerically optimized Taylor rule performs relatively worse as it does not react to rational and boundedly rational expectations (and the shock) while the inflation-targeting rule (22) does. Hence, it seems that reacting to the contemporaneous values of aggregate variables works relatively well only when *fractions are equal*.

α	CD^O	CD^{IT}	$var(\pi)^O$	$var(\pi)^{IT}$
0.95	1.904	1.104	0.580	0.590
0.9	4.071	2.567	0.597	0.618
0.7	17.805	12.700	0.664	0.750
0.5	39.071	25.812	0.670	0.833

Table 4: Consumption dispersion (CD) $var_i(c_t(i))$ under the optimal (O) interest rate rule (28) and the non-optimal inflation-targeting (IT) rule (22) for different fractions of rational forecasters α.

The trade-off between stabilizing inflation and consumption dispersion in this model becomes more evident when considering Figure 3. Figure 3 depicts consumption dispersion, the inflation variance (right ordinate) and absolute welfare losses (left ordinate) in case of a simple Taylor rule against different values of the coefficient on inflation. The higher the coefficient on inflation, the lower inflation volatility and the higher consumption dispersion. Therefore, it is not surprising that this also implies a local trade-off between minimizing welfare losses and reducing consumption dispersion.[15] Minimizing welfare losses requires the central bank to get a tight grip on inflation causing a substantial drop of individual consumption over time. As both agents react quite differently to the increase in the policy rate, as discussed in Section 4, consumption dispersion increases with the central bank's aggressiveness towards inflation.

Another observation worth mentioning is that consumption dispersion is minimized when the coefficient on inflation in the Taylor rule is one, i.e. when $i_t = \pi_t$ holds. This was analytically shown by Hagenhoff and Lustenhouwer (2019) in a model with fully rational agents and boundedly rational agents similar to this paper. Thus, the appearance of the same finding in this model serves as a robustness check for Hagenhoff and Lustenhouwer (2019).

[15] On the left side of the minimum, welfare losses decrease because of decreasing inflation. At the same time, consumption dispersion increases. At some point, however, welfare losses increase again as output volatility (not shown in Figure 3) becomes substantially higher as the central bank needs to contract output further to achieve further reductions in inflation. Therefore, the trade-off between welfare and consumption dispersion arises only locally, i.e. on the left side of the minimum where welfare losses and inflation decrease simultaneously.

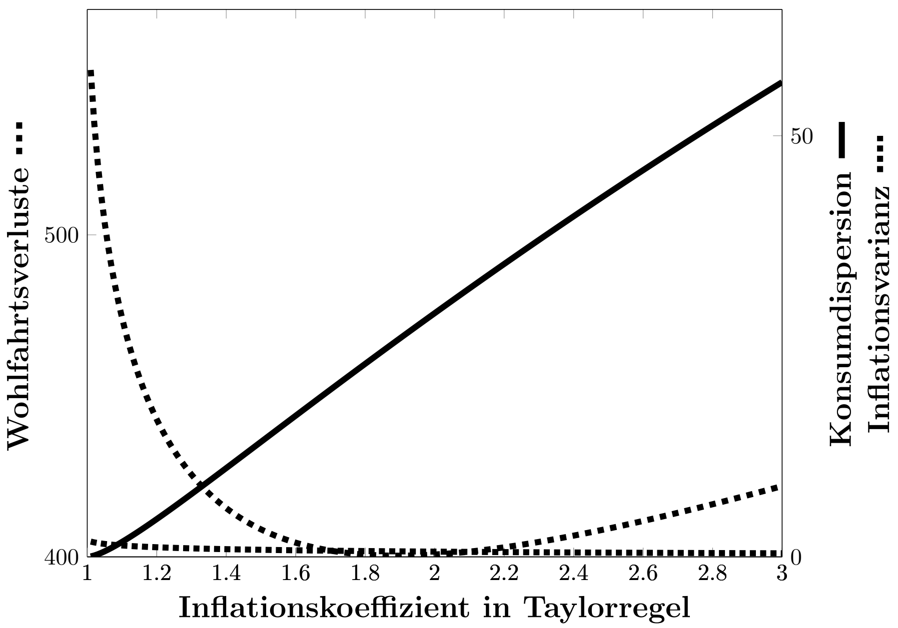

Figure 3: Trade-off between minimizing welfare, inflation and consumption dispersion under a simple Taylor rule and 70% of rational agents.

6 Conclusion

In this paper, I propose an optimal interest rate rule under heterogeneous expectations where the central bank commits to its policy from a timeless perspective. This rule incorporates the more complex nature of price dispersion and consumption dispersion under heterogeneous expectations as identified by Di Bartolomeo et al. (2016). Further, this rules performs considerably better than a microfounded version of the interest rate rule as in Gasteiger (2014). The implementation of the non-optimal inflation-targeting rule already requires an increase of 14.5 percent of steady-state consumption to compensate for the higher welfare losses relative to the optimal interest rate rule when only ten percent of the population form (naive) backward-looking expectations.

I additionally explore the properties of the expectations operator of "rational" agents in the Branch and McGough (2009) framework and find that the consumption Euler equation that includes model-consistent individual consumption expectations as in Di Bartolomeo et al. (2016) is inconsistent with the higher-order beliefs assumption of Branch and McGough (2009). This assumption puts a specific (non-rational) structure on higher-order beliefs which implies that not even "rational forecasters" understand the micro level fully. Therefore, I derive consumption decisions that account for this particular assumption which makes it the implementation of the optimal commitment policy by an interest rate rule possible in the first place.

Finally, I illustrate that the model implies a local trade-off between maximizing welfare and reducing consumption dispersion. The reason is that consumption dispersion increases with the central bank's aggressiveness towards inflation, as rational and boundedly rational forecasters' consumption decisions become more unequal with more aggressive inflation-targeting. Because inflation is the most important determinant of welfare, the central bank has to allow for a certain heterogeneity in consumption to maximize welfare.

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A Implementation under the conventional inflation-targeting objective

The policy problem under commitment and the conventional inflation-targeting objective is given by

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \frac{1}{2} \left[\left(\eta + \frac{1}{\sigma} \right) y_{t+s}^2 + \epsilon^2 \eta \delta \pi_{t+s}^2 \right] \tag{34}$$

$$+ \lambda_{t+s}[\pi_{t+s} - \alpha \beta E_t \pi_{t+s+1} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}]$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}}: \ E_t \left\{ \epsilon^2 \eta \delta \pi_{t+s} + \frac{\lambda_{t+s}}{2} - (1-\alpha)\beta^2\theta^2 \frac{\lambda_{t+s+1}}{2} - \alpha \frac{\lambda_{t+s-1}}{2} \right\} \stackrel{!}{=} 0 \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}}: \ E_t \left\{ \beta^s \left[\left(\eta + \frac{1}{\sigma} \right) y_{t+s} - \frac{\kappa}{2} \lambda_{t+s} \right] \right\} \stackrel{!}{=} 0. \tag{36}$$

Combining and solving for inflation gives

$$\pi_t = -\frac{1+\eta\sigma}{\sigma\epsilon^2\eta\delta\kappa}[y_t - \alpha y_{t-1} - (1-\alpha)\beta^2\theta^2 E_t y_{t+1}] \tag{37}$$

where the index s was dropped as the central bank employs timeless commitment. Combining with the Phillips and IS curve yields (22).

B Optimal monetary policy

B.1 Rewriting the model-consistent loss function

The period loss function L_t is given by

$$\begin{aligned} L_t = & \frac{\sigma\eta+1}{\sigma} y_t^2 + \frac{\alpha(y_t - c_t^R)^2}{(1-\alpha)\sigma} \\ & + \epsilon^2\eta\delta \left\{ \pi_t^2 + \frac{\xi_p(1-\alpha)}{\alpha} \left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa y_t - \frac{\alpha\kappa(y_t - c_t^R)}{(1+\eta\sigma)(1-\alpha)} \right]^2 \right\}. \end{aligned} \tag{38}$$

which can be rewritten using market clearing to eliminate c_t^B as

$$\begin{aligned} L_t = & \frac{\sigma\eta+1}{\sigma} y_t^2 + \frac{\alpha(y_t - c_t^R)^2}{(1-\alpha)\sigma} \\ & + \epsilon^2\eta\delta \left\{ \pi_t^2 + \frac{\xi_p(1-\alpha)}{\alpha} \left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa y_t - \frac{\alpha\kappa(y_t - c_t^R)}{(1+\eta\sigma)(1-\alpha)} \right]^2 \right\}. \end{aligned} \tag{39}$$

By multiplying out, we get

$$\begin{aligned} L_t = & \Gamma_1 y_t^2 + \Gamma_2 \pi_t^2 + \Gamma_3 \pi_{t-1}^2 + \Gamma_4 (c_t^R)^2 \\ & + \Gamma_5 y_t c_t^R + \Gamma_6 \pi_t c_t^R + \Gamma_7 \pi_{t-1} c_t^R + \Gamma_8 \pi_t \pi_{t-1} + \Gamma_9 \pi_t y_t + \Gamma_{10} \pi_{t-1} y_t \end{aligned} \tag{40}$$

with

$$\Gamma_1 = \frac{((\alpha-1)\eta\sigma-1)(\alpha(\eta^2\sigma^2(\delta\epsilon^2\kappa^2\xi_p-1)-1-2\eta\sigma)-\delta\epsilon^2\eta\kappa^2\xi_p\sigma(1+\eta\sigma))}{(1-\alpha)\alpha\sigma(1+\eta\sigma)^2} \tag{41}$$

$$\Gamma_2 = \frac{\delta\epsilon^2\eta(\alpha+\xi_p-\alpha\xi_p)}{\alpha} \tag{42}$$

$$\Gamma_3 = \frac{(1-\alpha)\beta^2\delta\epsilon^2\eta\theta^4\xi_p}{\alpha} \tag{43}$$

$$\Gamma_4 = \frac{\alpha(1+\eta\sigma(2+\delta\epsilon^2\kappa^2\xi_p)+\eta^2\sigma^2)}{(1-\alpha)\sigma(1+\eta\sigma)^2} \tag{44}$$

$$\Gamma_5 = \frac{2(\alpha+2\alpha\eta\sigma+\alpha\eta^2\sigma^2(1-\delta\epsilon^2\kappa^2\xi_p)+\delta\epsilon^2\eta\kappa^2\xi_p\sigma(1+\eta\sigma))}{(\alpha-1)\sigma(1+\eta\sigma)^2} \tag{45}$$

$$\Gamma_6 = \frac{2\delta\epsilon^2\eta\kappa\xi_p}{1+\eta\sigma} \tag{46}$$

$$\Gamma_7 = -\frac{2\beta\delta\epsilon^2\eta\theta^2\kappa\xi_p}{1+\eta\sigma} \tag{47}$$

$$\Gamma_8 = \frac{2(\alpha-1)\beta\delta\epsilon^2\eta\theta^2\xi_p}{\alpha} \tag{48}$$

$$\Gamma_9 = \frac{2\delta\epsilon^2\eta\kappa\xi_p((\alpha-1)\eta\sigma-1)}{\alpha+\alpha\eta\sigma} \tag{49}$$

$$\Gamma_{10} = \frac{2\beta\delta\epsilon^2\eta\theta^2\kappa\xi_p(1+\eta\sigma(1-\alpha))}{\alpha+\alpha\eta\sigma}. \tag{50}$$

B.2 Optimal interest rate rule

The policy problem under full commitment takes the following form:

$$\begin{aligned}
\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg[& \Gamma_1 y_{t+s}^2 + \Gamma_2 \pi_{t+s}^2 + \Gamma_3 \pi_{t+s-1}^2 + \Gamma_4 (c_{t+s}^R)^2 \\
& + \Gamma_5 y_{t+s} c_{t+s}^R + \Gamma_6 \pi_{t+s} c_{t+s}^R + \Gamma_7 \pi_{t+s-1} c_{t+s}^R + \Gamma_8 \pi_{t+s}\pi_{t+s-1} + \Gamma_9 \pi_{t+s} y_{t+s} + \Gamma_{10} \pi_{t+s-1} y_{t+s} \\
& + \lambda_{1,t+s}[y_{t+s} - \alpha E_t y_{t+s+1} - (1-\alpha)\theta^2 y_{t+s-1} + \sigma[i_{t+s} - \alpha E_t \pi_{t+s+1} - (1-\alpha)\theta^2 \pi_{t+s-1}]] \\
& + \lambda_{2,t+s}[\pi_{t+s} - \alpha\beta E_t \pi_{t+s+1} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}] \\
& + \lambda_{3,t+s}[c_{t+s}^R - E_t y_{t+s+1} + \sigma(i_{t+s} - E_t \pi_{t+s+1})] \Bigg].
\end{aligned} \tag{51}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_1 y_{t+s} + \Gamma_5 c^R_{t+s} + \Gamma_9 \pi_{t+s} + \Gamma_{10} \pi_{t+s-1} + \lambda_{1,t+s} - \kappa \lambda_{2,t+s}] \\ - \beta^{s+1}(1-\alpha)\theta^2 \lambda_{1,t+s+1} - \beta^{s-1}[\alpha \lambda_{1,t+s-1} + \lambda_{3,t+s-1}] \Big\} \stackrel{!}{=} 0 \tag{52}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_2 \pi_{t+s} + \Gamma_6 c^R_{t+s} + \Gamma_8 \pi_{t+s-1} + \Gamma_9 y_{t+s} + \lambda_{2,t+s}] \\ + \beta^{s+1}[2\Gamma_3 \pi_{t+s} + \Gamma_7 c_{t+s+1} + \Gamma_8 \pi_{t+s+1} + \Gamma_{10} y_{t+s+1} - (1-\alpha)\theta^2 \sigma \lambda_{1,t+s+1} \\ - (1-\alpha)\beta\theta^2 \lambda_{2,t+s+1}] - \beta^{s-1}[\alpha\sigma\lambda_{1,t+s-1} + \alpha\beta\lambda_{2,t+s-1} + \sigma\ \lambda_{3,t+s-1}] \Big\} \stackrel{!}{=} 0 \tag{53}$$

$$\frac{\partial \mathcal{L}}{\partial c^R_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_4 c^R_{t+s} + \Gamma_5 y_{t+s} + \Gamma_6 \pi_{t+s} + \Gamma_7 \pi_{t+s-1} \\ + \lambda_{3,t+s}] \Big\} \stackrel{!}{=} 0 \tag{54}$$

$$\frac{\partial \mathcal{L}}{\partial i_{t+s}} : E_t \Big\{ \beta^s \sigma \lambda_{1,t+s} + \beta^s \sigma \lambda_{3,t+s} \Big\} \stackrel{!}{=} 0. \tag{55}$$

Again, the index s can be dropped assuming commitment from a timeless perspective. Using $\lambda_{3,t} = -\lambda_{1,t}$ the FOCs can equivalently be written as

$$2\Gamma_1 y_t + \Gamma_5 c^R_t + \Gamma_9 \pi_t + \Gamma_{10} \pi_{t-1} + \lambda_{1,t} - \kappa \lambda_{2,t} - (1-\alpha)\beta\theta^2 \lambda_{1,t+1} - \\ \beta^{-1}(\alpha - 1)\lambda_{1,t-1} \stackrel{!}{=} 0 \tag{56}$$

$$2\Gamma_2 \pi_t + \Gamma_6 c^R_t + \Gamma_8 \pi_{t-1} + \Gamma_9 y_t + \lambda_{2,t} \\ + 2\Gamma_3 \beta \pi_t + \Gamma_7 \beta c_{t+1} + \Gamma_8 \beta \pi_{t+1} + \Gamma_{10} \beta y_{t+1} - (1-\alpha)\beta\theta^2 \sigma \lambda_{1,t+1} \\ - (1-\alpha)\beta^2\theta^2 \lambda_{2,t+1} + \beta^{-1}(1-\alpha)\sigma\lambda_{1,t-1} - \alpha\lambda_{2,t-1} \stackrel{!}{=} 0 \tag{57}$$

$$2\Gamma_4 c^R_t + \Gamma_5 y_t + \Gamma_6 \pi_t + \Gamma_7 \pi_{t-1} - \lambda_{1,t} \stackrel{!}{=} 0. \tag{58}$$

Eliminating the Lagrange multipliers yields the reduced-form FOC

$$\Delta^c_1 \pi_t + \Delta^c_2 \pi_{t+1} + \Delta^c_3 \pi_{t+2} + \Delta^c_4 \pi_{t-3} + \Delta^c_5 \pi_{t-2} + \Delta^c_6 \pi_{t-1} + \Delta_7 y_t + \Delta^c_8 y_{t+1} \\ + \Delta^c_9 y_{t+2} + \Delta^c_{10} y_{t-2} + \Delta^c_{11} y_{t-1} + \Delta_{12} c^R_t + \Delta^c_{13} c^R_{t+1} + \Delta^c_{14} c^R_{t+2} + \Delta^c_{15} c^R_{t-2} + \Delta^c_{16} c^R_{t-1} \stackrel{!}{=} 0. \tag{59}$$

with

$$\Delta_1 = \Gamma_6 + \Gamma_9 + (1-\alpha)(-1+2\alpha)\beta\Gamma_6\theta^2 + 2\Gamma_2\kappa + 2\beta\Gamma_3\kappa \tag{60}$$

$$+ (\alpha-1)\beta\theta^2(\Gamma_7 + \beta(\Gamma_{10}+\Gamma_7) + \Gamma_7\kappa\sigma) \tag{61}$$

$$\Delta_2 = \beta(\Gamma_8\kappa + (\alpha-1)\theta^2(\beta(\Gamma_9 + (\alpha-1)\beta\Gamma_7\theta^2) + \Gamma_6(1+\beta+\kappa\sigma))) \tag{62}$$

$$\Delta_3 = (\alpha-1)^2\beta^3\Gamma_6\theta^4 \tag{63}$$

$$\Delta_4 = \frac{(\alpha-1)\alpha\Gamma_7}{\beta} \tag{64}$$

$$\Delta_5 = \frac{\alpha^2\Gamma_6 + \Gamma_7 + \Gamma_7\kappa\sigma - \alpha(\Gamma_6 + \Gamma_7 + \beta(\Gamma_{10}+\Gamma_7) + \Gamma_7\kappa\sigma)}{\beta} \tag{65}$$

$$\Delta_6 = \Gamma_{10} + \Gamma_7 - \alpha(\Gamma_6+\Gamma_9) + (-1+(3-2\alpha)\alpha)\beta\Gamma_7\theta^2 + \Gamma_8\kappa \tag{66}$$

$$- \frac{(\alpha-1)\Gamma_6(1+\kappa\sigma)}{\beta} \tag{67}$$

$$\Delta_7 = 2\Gamma_1 + \Gamma_5 + (1-\alpha)(2\alpha-1)\beta\Gamma_5\theta^2 + \Gamma_9\kappa \tag{68}$$

$$\Delta_8 = \beta(\Gamma_{10}\kappa + (\alpha-1)\theta^2(\Gamma_5 + \beta(2\Gamma_1+\Gamma_5) + \Gamma_5\kappa\sigma) \tag{69}$$

$$\Delta_9 = (\alpha-1)^2\beta^3\Gamma_5\theta^4 \tag{70}$$

$$\Delta_{10} = \frac{(\alpha-1)\alpha\Gamma_5}{\beta} \tag{71}$$

$$\Delta_{11} = \frac{\Gamma_5 + \Gamma_5\kappa\sigma - \alpha(\Gamma_5 + \beta(2\Gamma_1+\Gamma_5) + \Gamma_5\kappa\sigma)}{\beta} \tag{72}$$

$$\Delta_{12} = 2\Gamma_4 + \Gamma_5 - 2(\alpha-1)(2\alpha-1)\beta\Gamma_4\theta^2 + \Gamma_6\kappa) \tag{73}$$

$$\Delta_{13} = \beta(\Gamma_7\kappa + (\alpha-1)\theta^2(\beta\Gamma_5 + 2\Gamma_4(1+\beta+\kappa\sigma)) \tag{74}$$

$$\Delta_{14} = 2(\alpha-1)^2\beta^3\Gamma_4\theta^4 \tag{75}$$

$$\Delta_{15} = \frac{2(\alpha-1)\alpha\Gamma_4}{\beta} \tag{76}$$

$$\Delta_{16} = -\alpha\Gamma_5 - \frac{2\Gamma_4(-1+\alpha+\alpha\beta+(\alpha-1)\kappa\sigma)}{\beta}. \tag{77}$$

Solving (59) for π_t and setting it equal to the NK Phillips curve yields

$$\begin{aligned} y_t = -\frac{1}{\Delta_7 + \Delta_1\kappa}((\alpha\beta\Delta_1 + \Delta_2)\pi_{t+1} + \Delta_3\pi_{t+2} + \Delta_4\pi_{t-3} + \Delta_5\pi_{t-2} \\ + (\Delta_6 + (1-\alpha)\beta\theta^2\Delta_1)\pi_{t-1} + \Delta_8 y_{t+1} + \Delta_9 y_{t+2} + \Delta_{10}y_{t-2} + \Delta_{11}y_{t-1} \\ + \Delta_{13}c^R_{t+1} + \Delta_{14}c^R_{t+2} + \Delta_{15}c^R_{t-2} + \Delta_{16}c^R_{t-1}) \end{aligned} \tag{78}$$

Setting (78) equal to the New IS curve, substituting c_t^R for consumption demand and solving for i_t gives the central bank's reaction function under commitment (28):

$$\begin{aligned} i_t =& \Omega_1 E_t\pi_{t+1} + \Omega_2 E_t\pi_{t+2} + \Omega_3\pi_{t-3} + \Omega_4\pi_{t-2} + \Omega_5\pi_{t-1} + \Omega_6 E_t y_{t+1} + \Omega_7 E_t y_{t+2} \\ &+ \Omega_8 y_{t-2} + \Omega_9 y_{t-1} + \Omega_{10} E_t c_{t+1}^R + \Omega_{11} E_t c_{t+2}^R + \Omega_{12} c_{t-2}^R + \Omega_{13} c_{t-1}^R + \Omega_{14} e_t \end{aligned} \tag{79}$$

with

$$\Omega_1 = \frac{\alpha\beta\Delta_1 + \Delta_2 + \sigma\Delta_{12} + \alpha\sigma(\Delta_7 + \Delta_1\kappa)}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{80}$$

$$\Omega_2 = \frac{\Delta_3}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{81}$$

$$\Omega_3 = \frac{\Delta_4}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{82}$$

$$\Omega_4 = \frac{\Delta_5}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{83}$$

$$\Omega_5 = \frac{\Delta_6 + (1-\alpha)\theta^2(\beta\Delta_1 + \sigma(\Delta_7 + \Delta_1\kappa))}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{84}$$

$$\Omega_6 = \frac{\Delta_{12} + \alpha\Delta_7 + \Delta_8 + \alpha\kappa\Delta_1}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{85}$$

$$\Omega_7 = \frac{\Delta_9}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{86}$$

$$\Omega_8 = \frac{\Delta_{10}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{87}$$

$$\Omega_9 = \frac{\Delta_{11} + (1-\alpha)\theta^2(\Delta_7 + \Delta_1\kappa)}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{88}$$

$$\Omega_{10} = \frac{\Delta_{13}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{89}$$

$$\Omega_{11} = \frac{\Delta_{14}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{90}$$

$$\Omega_{12} = \frac{\Delta_{15}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{91}$$

$$\Omega_{13} = \frac{\Delta_{16}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{92}$$

$$\Omega_{14} = \frac{\Delta_1}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)}. \tag{93}$$

B.3 Taking rational expectations as given

Defining $f_{t+s} = E_t y_{t+s+1}$, $g_{t+s} = E_t \pi_{t+s+1}$ and $h_{t+s} = E_t c^R_{t+s+1}$, the policy problem is

$$
\begin{aligned}
\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg[& \Gamma_1 y^2_{t+s} + \Gamma_2 \pi^2_{t+s} + \Gamma_3 \pi^2_{t+s-1} + \Gamma_4 (c^R_{t+s})^2 \\
& + \Gamma_5 y_{t+s} c^R_{t+s} + \Gamma_6 \pi_{t+s} c^R_{t+s} + \Gamma_7 \pi_{t+s-1} c^R_{t+s} + \Gamma_8 \pi_{t+s} \pi_{t+s-1} + \Gamma_9 \pi_{t+s} y_{t+s} + \Gamma_{10} \pi_{t+s-1} y_{t+s} \\
& + \lambda_{1,t+s}[y_{t+s} - \alpha f_{t+s} - (1-\alpha)\theta^2 y_{t+s-1} + \sigma[i_{t+s} - \alpha g_{t+s} - (1-\alpha)\theta^2 \pi_{t+s-1}]] \\
& + \lambda_{2,t+s}[\pi_{t+s} - \alpha\beta g_{t+s} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}] \\
& + \lambda_{3,t+s}[c^R_{t+s} - h_{t+s} + \sigma(i_{t+s} - g_{t+s})] \Bigg].
\end{aligned}
\tag{94}
$$

The first order conditions are

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y_{t+s}} : E_t \Big\{ & \beta^s [2\Gamma_1 y_{t+s} + \Gamma_5 c^R_{t+s} + \Gamma_9 \pi_{t+s} + \Gamma_{10} \pi_{t+s-1} + \lambda_{1,t+s} - \kappa \lambda_{2,t+s}] \\
& - \beta^{s+1} (1-\alpha)\theta^2 \lambda_{1,t+s+1} \Big\} \stackrel{!}{=} 0
\end{aligned}
\tag{95}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : \ E_t \Big\{ & \beta^s [2\Gamma_2 \pi_{t+s} + \Gamma_6 c^R_{t+s} + \Gamma_8 \pi_{t+s-1} + \Gamma_9 y_{t+s} + \lambda_{2,t+s}] \\
& + \beta^{s+1} [2\Gamma_3 \pi_{t+s} + \Gamma_7 c_{t+s+1} + \Gamma_8 \pi_{t+s+1} + \Gamma_{10} y_{t+s+1} - (1-\alpha)\theta^2 \sigma \lambda_{1,t+s+1} \\
& - (1-\alpha)\beta\theta^2 \lambda_{2,t+s+1}] \Big\} \stackrel{!}{=} 0
\end{aligned}
\tag{96}
$$

$$
\frac{\partial \mathcal{L}}{\partial c^R_{t+s}} : \ E_t \Big\{ \beta^s [2\Gamma_4 c^R_{t+s} + \Gamma_5 y_{t+s} + \Gamma_6 \pi_{t+s} + \Gamma_7 \pi_{t+s-1} + \lambda_{3,t+s}] \Big\} \stackrel{!}{=} 0
$$

$$
\frac{\partial \mathcal{L}}{\partial i_{t+s}} : \ E_t \Big\{ \beta^s \sigma \lambda_{1,t+s} + \beta^s \sigma \lambda_{3,t+s} \Big\} \stackrel{!}{=} 0.
\tag{97}
$$

Since the central bank acts under timeless commitment, the index s can be dropped. Using $\lambda_{3,t} = -\lambda_{1,t}$ the FOCs can equivalently be written as

$$2\Gamma_1 y_t + \Gamma_5 c_t^R + \Gamma_9 \pi_t + \Gamma_{10}\pi_{t-1} + \lambda_{1,t} - \kappa\lambda_{2,t} - (1-\alpha)\beta\theta^2 E_t\lambda_{1,t+1} \stackrel{!}{=} 0 \tag{98}$$

$$\begin{aligned} &2\Gamma_2\pi_t + \Gamma_6 c_t^R + \Gamma_8\pi_{t-1} + \Gamma_9 y_t + \lambda_{2,t} + 2\Gamma_3\beta\pi_t + \Gamma_7\beta E_t c_{t+1}^R + \Gamma_8\beta E_t\pi_{t+1} \\ &+ \Gamma_{10}\beta E_t y_{t+1} - (1-\alpha)\beta\theta^2\sigma E_t\lambda_{1,t+1} - (1-\alpha)\beta^2\theta^2 E_t\lambda_{2,t+1} \stackrel{!}{=} 0 \end{aligned} \tag{99}$$

$$2\Gamma_4 c_t^R + \Gamma_5 y_t + \Gamma_6\pi_t + \Gamma_7\pi_{t-1} - \lambda_{1,t} \stackrel{!}{=} 0. \tag{100}$$

Eliminating the Lagrange multipliers yields the reduced-form FOC

$$\begin{aligned} &\Delta_1\pi_t + \Delta_2 E_t\pi_{t+1} + \Delta_3 E_t\pi_{t+2} + \Delta_4\pi_{t-1} + \Delta_5 y_t + \Delta_6 E_t y_{t+1} + \Delta_7 E_t y_{t+2} \\ &+ \Delta_8 c_t^R + \Delta_9 E_t c_{t+1}^R + \Delta_{10} E_t c_{t+2}^R \stackrel{!}{=} 0 \end{aligned} \tag{101}$$

with

$$\Delta_1 = -\frac{\Gamma_6 + \Gamma_9 + 2\Gamma_2\kappa + 2\beta\Gamma_3\kappa - (1-\alpha)\beta\theta^2(\Gamma_7 + \beta(\Gamma_{10} + \Gamma_7) + \Gamma_7\kappa\sigma)}{\kappa} \tag{102}$$

$$\Delta_2 = \frac{(1-\alpha)\beta\theta^2(\beta(\Gamma_9 - (1-\alpha)\beta\Gamma_7\theta^2) + \Gamma_6(1+\beta+\kappa\sigma))}{\kappa} - \beta\Gamma_8 \tag{103}$$

$$\Delta_3 = -\frac{(\alpha-1)^2\beta^3\theta^4\Gamma_6}{\kappa} \tag{104}$$

$$\Delta_4 = -\frac{\Gamma_{10} + \Gamma_7 + \Gamma_8\kappa}{\kappa} \tag{105}$$

$$\Delta_5 = -\frac{2\Gamma_1 + \Gamma_5 + \Gamma_9\kappa}{\kappa} \tag{106}$$

$$\Delta_6 = -\frac{\beta((\alpha-1)\beta(2\Gamma_1 + \Gamma_5)\theta^2 + \Gamma_{10}\kappa + (\alpha-1)\Gamma_5\theta^2(1+\kappa\sigma))}{\kappa} \tag{107}$$

$$\Delta_7 = -\frac{(\alpha-1)^2\beta^3\theta^4\Gamma_5}{\kappa} \tag{108}$$

$$\Delta_8 = -\frac{2\Gamma_4 + \Gamma_5 + \Gamma_6\kappa}{\kappa} \tag{109}$$

$$\Delta_9 = -\frac{\beta((\alpha-1)\beta\Gamma_5\theta^2 + \Gamma_7\kappa + 2(\alpha-1)\Gamma_4\theta^2(1+\beta+\kappa\sigma))}{\kappa} \tag{110}$$

$$\Delta_{10} = -\frac{2(\alpha-1)^2\beta^3\theta^4\Gamma_4}{\kappa}. \tag{111}$$

Solving (101) for π_t and setting it equal to the NK Phillips curve yields

$$y_t = -\frac{1}{\Delta_5 + \Delta_1\kappa}(\Delta_6 y_{t+1} + \Delta_7 y_{t+2} + (\Delta_2 + \alpha\beta\Delta_1)\pi_{t+1} + \Delta_3\pi_{t+2} + (\Delta_4 + (1-\alpha)\beta\theta^2\Delta_1)\pi_{t-1}$$
$$+ \Delta_8 c_t^R + \Delta_9 c_{t+1}^R + \Delta_{10} c_{t+2}^R + \Delta_1 e_t). \tag{112}$$

Setting (112) equal to the New IS curve, substituting c_t^R for consumption demand and solving for i_t gives the reaction function under the assumption that the central bank takes rational expectations as given

$$i_t = \Omega_1^* y_{t-1} + \Omega_2^* E_t y_{t+1} + \Omega_3^* E_t y_{t+2} + \Omega_4^* \pi_{t-1} + \Omega_5^* E_t \pi_{t+1} + \Omega_6^* E_t \pi_{t+2}$$
$$+ \Omega_7^* E_t c_{t+1}^R + \Omega_8^* E_t c_{t+2}^R + \Omega_9^* e_t \tag{113}$$

with

$$\Omega_1^* = \frac{(1-\alpha)\theta^2(\Delta_5 + \Delta_1\kappa)}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{114}$$
$$\Omega_2^* = \frac{\Delta_6 + \Delta_8 + \alpha(\Delta_5 + \Delta_1\kappa)}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{115}$$
$$\Omega_3^* = \frac{\Delta_7}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{116}$$
$$\Omega_4^* = \frac{\Delta_4 + (1-\alpha)\theta^2(\beta\Delta_1 + \sigma(\Delta_5 + \Delta_1\kappa))}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{117}$$
$$\Omega_5^* = \frac{\alpha\beta\Delta_1 + \Delta_2 + \sigma\Delta_8 + \alpha\sigma(\Delta_5 + \Delta_1\kappa)}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{118}$$
$$\Omega_6^* = \frac{\Delta_3}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{119}$$
$$\Omega_7^* = \frac{\Delta_9}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{120}$$
$$\Omega_8^* = \frac{\Delta_{10}}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{121}$$
$$\Omega_9^* = \frac{\Delta_1}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)}. \tag{122}$$

The Ω-coefficients are expressed in terms of the targeting rule coefficients for simplicity. Writing them in terms of the deep model parameters would yield in part far to big expression.

B.4 Tables

Table 5 shows the reaction coefficients in interest rate rule (28) computed under baseline calibration but with varying α. It can be seen that the central bank puts relatively high weights on inflation (expectations) and the shock. In general, the weights that are placed on lagged inflation increase in absolute value when the fraction of rational forecasters, α, decreases as the central bank puts a higher weight on backward-looking expectations. The opposite holds true for the one-period ahead rational expectations. For the homogeneous rational expectations case, $\alpha = 1$, all reaction coefficients associated with heterogeneous expectations vanish.

Ω_x	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 1$ (RE)
y_{t-2}	0.001	0.004	0.01	0.018	0.029	0
y_{t-1}	0.115	0.052	-0.016	-0.085	-0.153	-0.139
$E_t y_{t+1}$	-0.08	-0.02	0.045	0.111	0.175	0.160
$E_t y_{t+2}$	-0.005	-0.008	-0.009	-0.007	-0.003	0
π_{t-3}	0.012	0.028	0.034	0.029	0.013	0
π_{t-2}	-0.609	-0.502	-0.38	-0.241	-0.085	0
π_{t-1}	2.747	2.082	1.420	0.792	0.238	0
$E_t \pi_{t+1}$	-0.482	0.115	0.697	1.229	1.639	1.851
$E_t \pi_{t+2}$	0.103	0.064	0.033	0.012	0.001	0
$E_t c^R_{t-2}$	0	-0.003	-0.009	-0.017	-0.029	0
$E_t c^R_{t-1}$	0.004	0.013	0.022	0.031	0.041	0
$E_t c^R_{t+1}$	-0.004	-0.012	-0.021	-0.031	-0.040	0
$E_t c^R_{t+2}$	0.003	0.007	0.008	0.007	0.003	0
e_t	1.275	1.227	1.154	1.056	0.932	0.859

Table 5: Values of reaction coefficients Ω_x in the interest rate rule (28) for different values of the share of rational forecasters α.

C Implementation with model-consistent individual consumption expectations

The policy problem under commitment and the conventional Euler equation with model-consistent individual consumption expectations takes the following form:

$$\begin{aligned}
\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg[& \Gamma_1 y_{t+s}^2 + \Gamma_2 \pi_{t+s}^2 + \Gamma_3 \pi_{t+s-1}^2 + \Gamma_4 (c_{t+s}^R)^2 \\
& + \Gamma_5 y_{t+s} c_{t+s}^R + \Gamma_6 \pi_{t+s} c_{t+s}^R + \Gamma_7 \pi_{t+s-1} c_{t+s}^R + \Gamma_8 \pi_{t+s}\pi_{t+s-1} + \Gamma_9 \pi_{t+s} y_{t+s} + \Gamma_{10}\pi_{t+s-1} y_{t+s} \\
& + \lambda_{1,t+s}[y_{t+s} - \alpha E_t y_{t+s+1} - (1-\alpha)\theta^2 y_{t+s-1} + \sigma[i_{t+s} - \alpha E_t \pi_{t+s+1} - (1-\alpha)\theta^2 \pi_{t+s-1}]] \\
& + \lambda_{2,t+s}[\pi_{t+s} - \alpha\beta E_t \pi_{t+s+1} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}] \\
& + \lambda_{3,t+s}[c_{t+s}^R - E_t c_{t+s+1}^R + \sigma(i_{t+s} - E_t \pi_{t+s+1})] \Bigg]. \qquad (123)
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y_{t+s}} : E_t \Big\{ & \beta^s [2\Gamma_1 y_{t+s} + \Gamma_5 c_{t+s}^R + \Gamma_9 \pi_{t+s} + \Gamma_{10}\pi_{t+s-1} + \lambda_{1,t+s} - \kappa\lambda_{2,t+s}] \qquad (124) \\
& - \beta^{s+1}(1-\alpha)\theta^2 \lambda_{1,t+s+1} - \beta^{s-1}\alpha\lambda_{1,t+s-1} \Big\} \stackrel{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : E_t \Big\{ & \beta^s [2\Gamma_2 \pi_{t+s} + \Gamma_6 c_{t+s}^R + \Gamma_8 \pi_{t+s-1} + \Gamma_9 y_{t+s} + \lambda_{2,t+s}] \\
& + \beta^{s+1}[2\Gamma_3 \pi_{t+s} + \Gamma_7 c_{t+s+1} + \Gamma_8 \pi_{t+s+1} + \Gamma_{10} y_{t+s+1} - (1-\alpha)\theta^2\sigma\lambda_{1,t+s+1} \qquad (125) \\
& - (1-\alpha)\beta\theta^2\lambda_{2,t+s+1}] - \beta^{s-1}[\alpha\sigma\lambda_{1,t+s-1} + \alpha\beta\lambda_{2,t+s-1} + \sigma\ \lambda_{3,t+s-1}] \Big\} \stackrel{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_{t+s}^R} : E_t \Big\{ & \beta^s [2\Gamma_4 c_{t+s}^R + \Gamma_5 y_{t+s} + \Gamma_6 \pi_{t+s} + \Gamma_7 \pi_{t+s-1} \qquad (126) \\
& + \lambda_{3,t+s}] - \beta^{s-1}\lambda_{3,t+s-1} \Big\} \stackrel{!}{=} 0
\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial i_{t+s}} : E_t \Big\{ \beta^s \sigma \lambda_{1,t+s} + \beta^s \sigma \lambda_{3,t+s} \Big\} \stackrel{!}{=} 0. \qquad (127)$$

Again, the index s can be dropped assuming commitment from a timeless perspective. Using $\lambda_{3,t} = -\lambda_{1,t}$ the FOCs can equivalently be written as

$$2\Gamma_1 y_t + \Gamma_5 c_t^R + \Gamma_9 \pi_t + \Gamma_{10}\pi_{t-1} + \lambda_{1,t} - \kappa\lambda_{2,t} - (1-\alpha)\beta\theta^2\lambda_{1,t+1} - \\ \beta^{-1}\alpha\lambda_{1,t-1} \stackrel{!}{=} 0 \tag{128}$$

$$2\Gamma_2\pi_t + \Gamma_6 c_t^R + \Gamma_8\pi_{t-1} + \Gamma_9 y_t + \lambda_{2,t} \\ + 2\Gamma_3\beta\pi_t + \Gamma_7\beta c_{t+1} + \Gamma_8\beta\pi_{t+1} + \Gamma_{10}\beta y_{t+1} - (1-\alpha)\beta\theta^2\sigma\lambda_{1,t+1} \\ - (1-\alpha)\beta^2\theta^2\lambda_{2,t+1} + \beta^{-1}(1-\alpha)\sigma\lambda_{1,t-1} - \alpha\lambda_{2,t-1} \stackrel{!}{=} 0 \tag{129}$$

$$2\Gamma_4 c_t^R + \Gamma_5 y_t + \Gamma_6\pi_t + \Gamma_7\pi_{t-1} - \lambda_{1,t} + \beta^{-1}\lambda_{1,t-1} \stackrel{!}{=} 0.. \tag{130}$$

(130) can be used to replace $\lambda_{1,t-1}$ and $\lambda_{1,t+1}$ with $\lambda_{1,t}$ in (128). Then, solving (128) for $\lambda_{1,t}$ and inserting in (129) yields a second-order difference equation in $\lambda_{2,t}$. A solution to this equation can in principle be substituted back into the difference equation, which would give a targeting rule. However, this solution is fairly complicated in which some parameter terms exponentially depend on time. The solution is available upon request. The resulting targeting rule and, hence, a reaction function would also be of such a complicated form where parameters exponentially depend on time. Consequently, no interpretable interest rate rule under commitment can be derived in this case.