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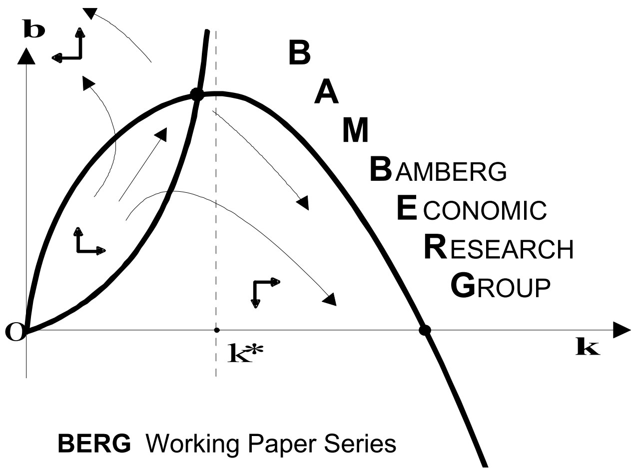

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Abstract

The research question relates to the quantitative impact of government bond purchases of the European Central Bank on inflation and other economic variables at the zero lower bound. At the core is a standard New Keynesian Dynamic Stochastic General Equilibrium model with several financial frictions. The model replicates the intended effect of Quantitative Easing regarding the drop in the government bond yield at the expense of a rise in public debt, and displays the crowding out effect on the balance sheet of the bank which spurs credit and output. Amid lower levels of wages and consumption, the overall quantitative effect is nevertheless not inflationary but deflationary. After a shock to the credit supply, Quantitative Easing is activated more if the zero lower bound on the policy rate is in place. Output after the first period, consumption as well as wages and inflation drop more in the case of the zero lower bound and Quantitative Easing does not make up for the loss. The same findings for the economic performance marked by these four variables are obtained for the analysis at the zero lower bound when a shock hits the exposure of financial intermediaries to public debt.

Keywords: Quantitative Easing, Taylor Rule, Zero Lower Bound, Moral Hazard

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1 Introduction

For more than a decade starting with the financial crisis 2007/2008, the purchase of sovereign bonds has formed part of the toolkit of the major central banks the Federal Reserve (Fed), the European Central Bank (ECB) and the Bank of Japan. This marks a territorial shift in the conduct of monetary policy in the Eurozone. The data of the stock of sovereign debt purchased by the ECB under the Public Sector Purchasing Program (PSPP) is revealing, which is gauged at more than EUR 2400 bn. around mid 2021 (ECB (2021)). Despite the break with the tradition of the Bundesbank (Mishkin (2010)) and the accompanied, fundamental divergence in the conduct of monetary policy across the members of the Eurozone (Braunberger (2021)), as well as the outline of criteria of the German Constitutional Court in May 2020 for monetary financing of the ECB to occur (Bundesverfassungsgericht (2020)), the conduct of sovereign bond purchases has been continued and reached new heights in the Covid pandemic through the Pandemic Emergency Purchasing Program (PEPP).

Quantitative Easing (QE) in the realm of Dynamic Stochastic General Equilibrium (DSGE) modeling is discussed in Cúrdia and Woodford (2011), Gertler and Karadi (2011), Kiley and Sim (2014), Wu and Zhang (2019), Debortoli et al. (2019) and Sims and Wu (2019) cit. in Taylor (2019). In Cúrdia and Woodford (2011), the balance sheet of the Fed is modelled and unconventional monetary policy amounts to direct lending to the private sector which reduces the resource costs for banks arising from the financial intermediation and eases the credit spread. In the model of Gertler and Karadi (2011), the fraction of assets intermediated by the central bank is determined by the rise of the credit spread in the event of a crisis. The costly enforcement or moral hazard problem is introduced for the financial firm which evaluates the incentives for the banker to supply or divert assets, putting a constraint on the balance sheet and henceforth the exposure to leverage. Based on the costly enforcement problem of Gertler and Karadi (2011) and (2013), Sims and Wu (2019) model QE by the purchase of both private and exogenously fixed public bonds with the creation of reserves, thereby relaxing the endogenous leverage constraint on financial intermediaries. Kiley and Sim (2014) compare public intervention similar to Gertler and Karadi (2011) by purchasing capital assets from financial intermediaries at market or subsidized prices with the one of direct capital injections. In the shadow rate New Keynesian model of Wu and Zhang (2019), QE is introduced through an inverse relation between the risk premium in the market and bonds purchased by the central bank, where the risk premium is the difference between the yield on bonds and the Fed funds rate. The comparison is made to a shadow interest rate described by the Taylor rule.

The zero and effective lower bounds on the policy rate (ZLB and ELB, respectively) have naturally regained much attention in macroeconomic analysis following the global financial crisis, in the aftermath of which the Fed and the ECB both reached the constraint. Since then, the ECB had not succeeded to escape from the lower bound on the interest rate until July 2022 and it was quickly reinforced for the Fed when the COVID-19 pandemic struck. Only with inflation vastly above the target in the USA and the Eurozone, the two major central banks started to raise policy rates with the ECB lagging far behind the Fed. Correspondingly, New Keynesian models with the ZLB or ELB in place are found in Sims and Wu (2019) and Mertens and Williams (2019) both referred to in Taylor (2019), Debortoli et al. (2019), Wu and Zhang (2019), as well as in the FRB/US model analyzed by Bernanke et al. (2019*b*) cit. in Taylor (2019). In the interplay with fiscal policy, Christiano (2011) cit. in Van der Kwaak and Van Wijnbergen (2017) finds increased effectiveness of fiscal stimuli marked by a significantly higher multiplier at the ZLB. Taylor (2019) draws on the importance of the ELB on the interest rate to reason for a rules-based monetary system, both nationally and emerging from it internationally, and QE included as an instrument in a rule. In the model of Sims and Wu (2019) referred to in Taylor (2019), the ZLB is implemented for the deposit rate while a binding reserve requirement for banks drives the interest rate below the deposit rate and potentially below zero. Mertens and Williams (2019) referred to in Taylor (2019) apply the standard New Keynesian model to compare different monetary policy rules when policy is constrained by the lower bound on the policy rate. The flexible price level targeting as a monetary policy rule, in comparison, is not ranked among the best performing in the analysis of Bernanke et al. (2019*b*) with Bernanke et al. (2019*a*) referred to in Taylor (2019). The authors apply the FRB/US model to study 10 policy rules at the ELB and low neutral interest rates in the scenarios of completely model consistent expectations as well as consistent expectations only by asset market participants. A related caveat based on Wieland (2018) is that the level of the neutral interest rate is not as clear cut and easy to determine as policy makers and macro researchers alike presume. Debortoli et al. (2019) test for the irrelevance hypothesis of the ZLB by looking at the volatility of macro variables firstly by way of regressions and secondly in the standard, simple New Keynesian model responding to a demand shock. The authors conclude that the closer description of the data by the shadow rate is due to the incorporation of unconventional monetary policy getting around the ZLB without drawing a distinction between forward guidance and QE. Likewise the cited model of Wu and Zhang (2019), the shadow rate captures the unconventional monetary policies at the ZLB without imposing a structural break at the ZLB.

The paper approaches the topic of QE in a distinctive way within a New Keynesian DSGE model with financial frictions at play. One unique characteristic of the model is the combination of several frictions within one comprising model: a bank lending channel introducing capital requirements and a principal-agent problem between financial intermediaries and intermediate goods producers, both modeled as monitoring costs similar to Cúrdia and Woodford (2011) cit. in Alpanda et al. (2014), costs associated with the market based monitoring of the public entity as well capital adjustment costs. The main contribution of the paper is the modeling of QE with the moral hazard problem embedded. In comparison to the cited DSGE literature, the purchase of government bonds marks the unconventional monetary policy of the ECB of QE, not direct lending to the private sector. The stock of sovereign debt evolves endogenously and the effects spill over to the budget constraint of the sovereign. Essentially, the model is built on different interest rates for deposits and long term government bonds, each targeted by different policy actions of the central bank. The targeting of the long term bonds not only allows for a more detailed exploration of the intersection of monetary with fiscal spheres, it also makes possible the construction of the yield curve and the goal of monetary policy of controlling the longer end of it. Under the regime of monetary financing, the central bank conducts QE by the purchase of the optimal quantity of government bonds in its maximization of its seignorage given the budget constraint of the government. In other words, the modeling does feature an optimization structure for the bond purchases which is unique in the literature. Besides the endogenous determination, a Taylor type rule describes QE in reaction to the gaps in inflation and output borrowed from Cúrdia and Woodford (2011) and Sims and Wu (2019). In the optimal decision making, the model furthermore features an element of Moral Hazard as delayed or missing political efforts to reduce the excessive debt levels in several countries of the Eurozone and the general suppression of market forces by the ECB. In line with the overall assumption in the DSGE literature of the deposit rate to constitute the policy rate of the monetary authority, the lower bound is enforced by the deposit facility and refers to the short term rate. Another unique feature of the model is the parallel implementation of a second lower bound, i.e. on the lending rate, taking shape in the economic response to the credit shock.

The partly estimated model replicates the intended effect of QE regarding the drop in the government bond yield at the expense of a rise in public debt, and displays the crowding out effect on the balance sheet of the bank which spurs credit and output. Amid lower levels of wages and consumption, the overall quantitative effect is nevertheless not inflationary but deflationary. After a shock to the credit supply, QE is activated more if the ZLB on the policy rate is in place. Output after the first period, consumption as well as wages and inflation drop more in the case of the ZLB and QE does not make up for the loss. The same findings for the economic performance marked by these four variables are obtained for the analysis at the ZLB when a shock hits the exposure of financial intermediaries to public debt.

The remainder of the paper is organized as follows: Section 2 outlines the model with respect to the agents of households, final and intermediate goods producers, financial intermediaries and the public sector. Section 3 elaborates on the core topic of QE by setting the stage with a short summary of the theoretical underpinnings against QE and by introducing it analytically into the model. Finally, the dynamics in the model associated with a ramp up of QE are discussed. Section 4 introduces the ZLB in the model. Section 5 lists the parameter and the steady state values. Section 6 conducts policy analysis in the scenarios of a productivity shock, a monetary shock, and shocks both to the credit supply and to the public bond exposure at the financial intermediary. Section 7 concludes.

2 Model

The standard DSGE model is similar to the one developed in Ireland (1997) cit. in Pichler (2007), and subsequently in Pichler (2008). The economy is populated by households, intermediate goods and final goods producers as well as the central bank. Conventional monetary policy is described by the standard Taylor rule. The simplifying assumption of a representative agent for each of the actors in the economy applies. The intermediate goods producers face monopolistic competition in the markets of the respective inputs and price adjustment costs. The households incur capital adjustment costs in the investment decision. Households provide the funding to banks via deposits and banks provide credit to the intermediate goods producer for them to finance their activities. Four financial frictions form part of the model: a bank lending channel and a principal-agent problem between financial intermediaries and intermediate goods producers, both modeled as monitoring costs similar to Cúrdia and Woodford (2011) cit. in Alpanda et al. (2014)[1], costs associated with the market based monitoring of the public entity as well equity capital adjustment costs. The public sector levies taxes and faces an exogenous stream of spending needs. A fiscal rule for taxes with the counterpart of transfers to the household balances the budget, which includes a distinct term for the incentives problem of QE driving up the public debt. In a response to a deviation in output, countercyclical fiscal policy is at play.

2.1 Households

The representative household maximizes the utility function with respect to labor and consumption

$$\max U = E_0 \sum_{t=0}^{T} \beta^t [\frac{c_t^{1-\tau} - 1}{1-\tau} + \chi_h (1 - h_t)], \tag{1}$$

where β is the discount factor, c is consumption and h is hours of labor, the subscript indicates the time period. χ_h is the parameter for the weight of preference given to leisure time as opposed to working time. The utility function is concave in its element consumption with c_t in the environment of its steady state value and $\tau = 2$, while it is linear in leisure. Unlike in Pichler (2008), utility is not derived from money holdings because in the respective budget constraint of the household it is a mere accounting identity with a corresponding lump sum transfer. In the standard New Keynesian Model, hence, money supply does not spill over to inflation.

The budget constraint is:

$$w_t h_t + r_{k,t} k_t + \frac{r_{s,t-1} s_{t-1}}{\pi_t} + tr_t + d_{f,t} + d_{e,t} - t_t - c_t - x_t - q_{k,t} - s_t (1 + \mu)_t \geq 0, \tag{2}$$

[1] The journal article of the model for the Canadian economy is published in Alpanda et al. (2018)

w is the wage, $r_{k,t}$ is the rent on capital k, the law of motion for capital depends on investment $x_t = k_{t+1} - (1-\delta)k_t$ where δ is the depreciation rate of capital at t. γ is the cost parameter for the quadratic adjustment costs of capital $q_{k,t} = (\gamma/2)(x_t/k_t - \delta)^2 k_t$. $d_{e,t}$ and $d_{f,t}$ are the dividends of the firms and of the banks, respectively. Both dividends accrued by the households are reasoned by the ownership in aggregate. $r_{s,t}$ is the gross savings rate on the real market value of the deposits s_t. All variables are denoted in real terms. p_t is the price of final goods at time t, $\pi_t = p_t/p_{t-1}$ is the rate of inflation.

The monitoring costs have the following form similar to Cúrdia and Woodford (2011) cit. in Alpanda et al. (2014) and to Gambacorta and Signoretti (2014):

$$(1+\mu)_t = \chi_{s1}[\nu \frac{l_t + b_{f,t}}{k_{f,t}}]^{\chi_{s2}} shk_{s,t}. \tag{3}$$

ν is the capital requirement target defining a certain ratio of equity capital $k_{f,t}$ over the total assets of the financial intermediary. χ_{s1} is a level parameter accounting for the spread in the costs in the steady state, χ_{s2} an elasticity changing the costs with respect to the deviation from the capital requirement. $shk_{s,t}$ is an AR(1) process, as specified below.

A capital ratio below the target influences the decision of the household to deposit money with the bank: It increases the costs of deposits which constitutes an additional expense for the household. As it is created on the supply side via the financial sector and, in general equilibrium, affects the expected lending rate and thus the lending, it is named bank lending channel. The channelling through is analytically discussed below in the description of the banking sector. There is asymmetric information between the household and the bank with respect to the solvency of the bank, or at least to the quality of deposits and assets within the banking sector, for which reason monitoring costs incur. A lower equity capital ratio than required marks a higher perceived risk inherent in the financing and is associated with higher costs for the household: in equilibrium, the current savings rate is related to the monitoring costs. Finally, the household has an incentive that the bank is profitable because of accruing at least a part of it via distributed dividends, which remain after accounting for the reinvested profits by financial intermediaries. The household is aware of this by requiring the bank to fulfil the capital requirement and has some "skin in the game": the monitoring costs rise with a deviation from the target such that in equilibrium, the positioning of the banks with respect to the capital requirement influences the costs of their funding and thus the funding itself:

$$r_{s,t} = (1+\mu)_t E_t[\frac{c^{\tau}_{t+1}}{c^{\tau}_t} \frac{\pi_{t+1}}{\beta}]. \tag{4}$$

In this context, Alpanda et al. (2014) note: "Although this formulation abstracts from bank default per se, these monitoring costs can be interpreted as the fraction of funds that are defaulted upon by the banks (i.e., "bad loans"), following Curdia and Woodford (2011). Another interpretation of these monitoring costs is that they reflect the cost of purchasing default insurance on funds extended to banks, similar to a credit default swap (Amdur, 2010)." (Alpanda et al. (2014, p.10))

2.2 Final goods producer

The firms are split into producers of final and intermediate goods. The final goods producer is assumed to be perfectly competitive, i.e. all the profits are wiped out by competition. The final goods producer resorts to a constant returns-to-scale technology and use the intermediate goods $y_{j,t}$ as inputs where the intermediate goods producer are indexed by $j \in [0,1]$:

$$y_t = (\int_0^1 y_{j,t}^{\frac{\theta-1}{\theta}} dj)^{\frac{\theta}{\theta-1}}, \tag{5}$$

where θ is the elasticity of substitution between two intermediate inputs and y_t is the final output. Regarding the final good, the firm will not able to produce as much if it has to substitute one input with another. In other words, the reshuffling of inputs consumes resources. Likewise, the imperfect substitutability gives the intermediate goods producer market power because each intermediate good producer is a monopolist in the market of the single input and can set the price instead of taking it as given in a perfectly competitive market.

2.3 Intermediate goods producer

The representative agent maximizes the discounted value of expected real dividends $d_{e,t}$:

$$\max \mathbb{E}_0 \sum_{t=0}^{T} \beta_e^t \lambda_t d_{e,t}, \tag{6}$$

with β_e^t the time discount factor and λ_t the Lagrangian multiplier from the budget constraint of the households, in other words the shadow price of one unit of additional wealth. It gives the change in the value of the objective function, i.e. the maximization of the utility function, if the budget constraint is changed by one unit, i.e. by one unit of additional wealth. Since $\beta_e < \beta$ in calibration, the time discount factors differ slightly as in Iacoviello (2013) cit. in Alpanda et al. (2014). The actors weight the time preferences slightly different due to asymmetric information. The period-by-period stream of dividends,

$$d_{e,t} = \frac{p_{j,t} y_{j,t}}{p_t} + l_t - r_{k,t} k_{j,t} - w_t h_{j,t} - \frac{r_{l,t} l_{t-1}}{\pi_t} - q_{p,t}, \tag{7}$$

by which $p_{j,t}$ denotes the price of the intermediate goods. $r_{l,t}$ is the gross lending rate on loans at current prices, l_{t-1} is the real market value of the loans taken in the last period. The loans are used to finance the production with the inputs provided by the household. The household works for the intermediate goods producer, provides physical capital and receives in turn the wage and the rent, respectively. The intermediate goods producing firm exhibits the standard constant returns-to-scale Cobb-Douglas production function,

$$y_{j,t} = k_{j,t}^{\alpha}(shk_{z,t}h_{j,t})^{1-\alpha}. \tag{8}$$

$shk_{z,t}$ is a technology shock, as specified below. Moreover, the firms are subject to quadratic price adjustment costs $q_{p,t}$ in line with Rotemberg (1982). Price changes are costly while the changes are compared to the steady state of the inflation $\bar{\pi}$. The underlying assumption is that, varying from industry to industry, price changes are costly, as customer relationships are destructed or due to menu costs. The costs given in real terms, i.e. adjusted for the general price level in the economy p_t, are

$$q_{p,t} = \frac{\phi}{2}(\frac{p_{j,t}/p_{j,t-1}}{\bar{\pi}} - 1)^2 y_t. \tag{9}$$

The implied version of a New Keynesian Phillips Curve is of paramount importance for the inflation dynamics and the effectiveness of monetary policy in the standard New Keynesian model:

$$0 = \frac{1}{c_t^{\tau}}[1 - \theta + \theta\frac{w_t h_t}{(1-\alpha)y_t} - \phi(\frac{\pi_t}{\bar{\pi}} - 1)\frac{\pi_t}{\bar{\pi}}] + \beta_e E_t[\frac{1}{c_{t+1}^{\tau}}(\frac{\pi_{t+1}}{\bar{\pi}} - 1)\phi\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{\bar{y}}]. \tag{10}$$

It is forward-looking due to the price adjustment costs. It features an expression for the costs of production in terms of labor, a gap in inflation, and additionally the expected deviations of both inflation and output from their steady states.

2.4 Financial intermediaries

The financial intermediaries maximize the discounted value of expected dividends $d_{f,t}$ under the balance sheet constraint,

$$\max E_0 \sum_{t=0}^{T} \beta^t \lambda_t d_{f,t}, \tag{11}$$

whilst both the stochastic and the time discount factors are congruent with the ones of the shareholders. The balance sheet of a bank is comprised of the total exposure in credit l_t as well as sovereign bonds $p_{b,t}b_{f,t}$ on the asset side and the deposits of the household s_t and the equity capital $k_{f,t}$ on the liability side:

$$p_{b,t}b_{f,t} + l_t = s_t + k_{f,t}. \tag{12}$$

The period by period cash flow is given by:

$$d_{f,t} = \frac{r_{l,t}l_{t-1}}{pi_t} + \frac{r_{b,t}}{pi_t}p_{b,t-1}b_{f,t-1} + s_t - \frac{r_{s,t-1}s_{t-1}}{pi_t} - l_t(1+\zeta)_t - (1+\xi)_t p_{b,t}b_{f,t} - q_{k,f,t} - d_{k,f,t}. \quad (13)$$

The bank accrues profits from its business activities in the credit and the sovereign bond markets while it remunerates the household for providing the finance in the last period by the gross interest rate $r_{s,t-1}$. Due to the ZLB imposed, the interest paid does not move into negative territory. l_t refers to the total loan exposure, that is, the introduction of a price is obsolete in this context. Several financial frictions drive the dynamic system of the economy, on the side of the financial intermediary costs for monitoring both the entrepreneur and the government are incurred and it is not frictionless to adjust the level of equity capital by reinvesting a part of the profits.

Upfront the investment in sovereign debt is a term which displays the monitoring costs of banks with respect to the stance of public policy and solvency of the government,

$$(1+\xi)_t = \chi_{b1}(\frac{b_t}{\bar{b}})^{\chi_{b2}} shk_{b,f,t}, \quad (14)$$

where χ_{b1} is the level parameter, χ_{b2} is the elasticity and $shk_{b,f,t}$ an AR(1) process attached.[2] Attributable to the latter are other factors contributing to the monitoring costs than the deviation of the outstanding value of debt from its steady state, such as an exogenous risk shock. Moreover, the process adjusts for the variation around the ratio. In equilibrium, the expected, discounted, real yield on government bonds reacts positively to a debt level rising above its long term value. In other words, the financial intermediaries form expectations about the yield in accordance with the solvency of the sovereign and, as such, it is regarded as the underlying market mechanism in the model for driving the yield:

$$\beta E_t[(\frac{1}{c^{\tau}_{t+1}})(\frac{r_{b,t+1}}{\pi_{t+1}})] = \frac{1}{c^{\tau}_t}[(1+\xi)_t + \chi_{b2}\chi_{b1}(\frac{b_t}{\bar{b}})^{\chi_{b2}-1} shk_{b,f,t}(\frac{b_{f,t}}{\bar{b}})] - (\frac{1}{c^{\tau}_t}) + (\frac{1}{c^{\tau}_t})(1+\mu)_t. \quad (15)$$

The main determinant for the yield in the market runs against the determination of the central bank in the realm of its government bond purchases, as discussed in the section on QE.

$k_{f,t}$ is determined via reinvested profits whilst the remainder of profits are distributed to the households in the form of dividends. The accumulation of bank capital is assumed to follow the law of motion in the economy,

[2]The functional form is assumed to be in accordance with the other two monitoring costs imposed, hence, reference is made respectively to Cúrdia and Woodford (2011) cit. in Alpanda et al. (2014).

$$k_{f,t+1} = (1-\kappa)k_{f,t} + d_{k,f,t}, \tag{16}$$

whereas κ is the parameter evaluating the recurring consumption of bank capital and $d_{k,f,t}$ are the reinvested profits of the bank in the current period. In order to improve the capital base and thereby ease the monitoring costs of the households, the banks are instructed to reinvest the profits into equity. Obviously, an increase in reinvested profits cet. par. comes at the expense of another position in the cash flow.

Model simulations reveal that the responses of reinvested profits and bank dividends are very short lived in the construction of the model thus far and related variables exhibit unrealistic developments, especially with the Basel III macroprudential framework intended to establish a smooth transition to higher capital requirements. The sequential introduction of capital adjustment costs touch the very essence of capital requirements, that is, the funding via capital is more expensive for banks than via savings and banks are reluctant to hold more capital, as Kiley and Sim (2014) outline. Projected to the COVID-19 pandemic, the ECB requested of banks to retain the dividends, as Rasch (2021) reports, and Alpanda et al. (2014) note as well: "this feature is similar to Jermann and Quadrini (2012), and captures evidence in the corporate finance literature regarding the smoothing of corporate dividend payouts. It also ensures that banks cannot decrease dividends too much during recessions, and therefore the decline in their net worth cannot be fully cushioned by a corresponding decline in dividend payments." (Alpanda et al. (2014, p.14)) The adjustment costs are assumed to be of quadratic form,

$$q_{k,f,t} = \frac{o}{2}[(\frac{d_{k,f,t}}{k_{f,t}} - \kappa)^2]k_{f,t}. \tag{17}$$

$(1+\zeta)_t$ are the monitoring costs for the banks when contracting the loans with reference to Cúrdia and Woodford (2011) cit. in Alpanda et al. (2014),

$$(1+\zeta)_t = \chi_{l1}(\frac{l_t}{d_{e,t}})^{\chi_{l2}} shk_{l,t}. \tag{18}$$

These costs arise due to the monitoring of the leverage position of the borrower of the funds, that is, the intermediate goods producer. Thereby, an agency-cost problem is introduced with the notation of χ_{l1} for the level parameter and χ_{l2} for the elasticity. The AR(1) process $shk_{l,t}$ accounts for the variance in the net worth of the firm (Boivin et al. (2010) cit. in Alpanda et al. (2014)), and additionally for the risk inherent in the loan not reasoned by the leverage of the entrepreneur (Christiano et al. (2010) cit. in Alpanda et al. (2014)). Under the latter are subsumed, for instance, precautionary and risk-adverse motives of banks to hesitate in times of crises to supply credit and instead prefer to hoard the liquidity. Higher net worth of the intermediate producers thus lowers the monitoring costs which channels through to a lower expected price on credit with a feedback effect on the dividends, all else equal, in the cash flow of the entrepreneur. This, in turn, lowers further the monitoring costs and sets the spiral in motion. Due to the optimal decision making of the bank, the expected lending rate is a positive function of the monitoring costs involved:

$$\beta E_t(\frac{1}{c^{\tau}_{t+1}}\frac{r_{l,t+1}}{\pi_{t+1}}) = (\frac{1}{c^{\tau}_t})[\chi_{l2}(1+\zeta)_t + (1+\zeta)_t] - (\frac{1}{c^{\tau}_t}) + (\frac{1}{c^{\tau}_t})(1+\mu)_t. \tag{19}$$

Furthermore, it results in the finding of the bank capital channel that the capital position of the bank in dependence on the capital requirement finally has an impact on the lending rate. Higher monitoring costs of the household not only increase the savings rate but it channels through to a higher, expected lending rate.

In combination with the optimal choice with respect to savings, the monitoring costs drive a wedge between the expected discounted lending and the current funding rate for the banks: the more costs the banks bear due to the leverage of the entrepreneur, the higher is the expected spread:

$$\beta E_t[\frac{1}{c^{\tau}_{t+1}}\frac{r_{l,t+1} - r_{s,t}}{\pi_{t+1}}] = (\frac{1}{c^{\tau}_t})[\chi_{l2}(1+\zeta)_t + (1+\zeta)_t] - (\frac{1}{c^{\tau}_t}), \tag{20}$$

or alternatively solved for the discounted, expected lending rate:

$$\beta E_t[\frac{1}{c^{\tau}_{t+1}}\frac{r_{l,t+1}}{\pi_{t+1}}] = (\frac{1}{c^{\tau}_t})[\chi_{l2}(1+\zeta)_t + (1+\zeta)_t] - (\frac{1}{c^{\tau}_t}) + \beta E_t[\frac{1}{c^{\tau}_{t+1}}\frac{r_{s,t}}{\pi_{t+1}}]. \tag{21}$$

It becomes clear that the rate required depends on the monitoring costs incurred by the banks and on the costs of funding.

2.5 Public Sector

The single public entity issues long-term bonds and levies lump-sum taxes on the households to finance public expenditure. It is followed García-Cicco and Kirchner (2016), Sims and Wu (2019) and to a limited extent Van der Kwaak and Van Wijnbergen (2017) in that taxes constitute the fiscal instrument to adjust endogenously to stabilize the debt. Moreover, transfers to the household are in place to balance the cash flows. The demand for bonds arises on the side of both the financial intermediaries and the central bank, whilst the supply by the sovereign assures the equilibrium at a specific price and inversely related yield. Hence, these positions at the balance sheet of the sovereign adjust endogenously which is different to the simplifying assumption in Sims and Wu (2019) that the stock of outstanding government bonds is fixed in real terms, as such incapable to incorporate the distorted incentives of sovereign bond purchases for the evolution of sovereign debt and an elaborated, interfering public sector.

The budget constraint reads as follows:

$$0 = p_{b,t}b_t + t_t - g_t - tr_t - p_{b,t-1}b_{t-1}\frac{r_{b,t}}{\pi_t} + \chi_{qe1}(\frac{b_t}{\bar{b}})^{\chi_{qe2}}b_{cb,t} + d_{cb,t}, \tag{22}$$

tr_t are the transfers to the household, t_t the taxes and the total quantity of sovereign bonds b_t is split into holdings at the banks $b_{f,t}$ and at the central bank $b_{cb,t}$,

$$b_t = b_{f,t} + b_{cb,t}. \tag{23}$$

The seignorage accrued by the central bank $d_{cb,t} = p_{b,t-1}b_{cb,t-1}\frac{r_{b,t}}{\pi_t} - p_{b,t}b_{cb,t}$ is transferred to the government like in Gertler and Karadi (2011) and Sims and Wu (2019), such that the budget constraint simplifies to:

$$0 = p_{b,t}b_{f,t} + t_t - g_t - tr_t - p_{b,t-1}b_{f,t-1}\frac{r_{b,t}}{\pi_t} + \chi_{qe1}(\frac{b_t}{\bar{b}})^{\chi_{qe2}}b_{cb,t}. \tag{24}$$

It is thus abstained in the model from the reinvestment of the surplus in bonds or, as occurred in the case of Greece, that the interest payments are not collected and instead used for the redemption of the credit lines provided to the sovereign. It refers rather to the remaining seignorage distributed to the member countries in accordance with the capital key at the ECB. The term with χ_{qe1} upfront displays the distorted incentives out of QE and is discussed in the section on QE.

The bond pricing is borrowed from Woodford (1998, 2001) cit. in Van der Kwaak and Van Wijnbergen (2017). The quantity of bonds b_t times the the real price of bonds $p_{b,t}$ gives rise to the total real value of outstanding debt $p_{b,t}b_t$. The decay parameter $\rho_{p,b} \in [0;1]$ determines the maturity of outstanding debt and is specifically calibrated to correspond to the average maturity of the long-term bonds purchased in the QE programs. Constant coupon payments are assumed to be accrued every period yet with the decaying structure over time, that is, the coupon in $t+1$, the coupon times $\rho_{p,b}$ in $t+2$, the coupon times $\rho_{p,b}^2$ in $t+3$ and so on. Equivalent to rolling over the bond with $\rho_{p,b}$ every period, the geometric series reduces to the duration or average maturity of $1/(1-\beta\rho_{p,b})$ where β is the time discount factor. The comprising formula reveals that the gross yield on the bond $r_{b,t}$ evolves inversely to the bond price and is a function of the accruing coupon payments,

$$r_{b,t} = \frac{coupon + \rho_{p,b}p_{b,t}}{p_{b,t}}. \tag{25}$$

The advantage over the modeling in Christiano et al. (2014), in which the long term bond is merely an alternative investment for the household with clearing at $t+40$, is that bond pricing is introduced for long term bonds and that there are periodic coupon payments to construct the yield curve. The overall majority of DSGE literature abstains from long term sovereign bonds at all, instead the investment is limited to one period bonds or deposits as, for example, in Alpanda et al. (2014), García-Cicco and Kirchner (2016), Gertler and Karadi (2011), Cúrdia and Woodford (2011) and De Carvalho et al. (2014).

In principle, the response to the varying debt can be assigned to any fiscal instrument available. The model assumes the response by taxes whereas the stream of government expenditures is modelled as an AR(1) process,

$$log(\frac{g_t}{\bar{g}}) = rho_g log(\frac{g_{t-1}}{\bar{g}}) + \epsilon_{g,t}. \tag{26}$$

The fiscal rule ensures that a higher debt level in the previous period gets targeted by higher taxes in the current period, in log deviation from the steady state:

$$log(\frac{t_t}{\bar{t}}) = rho_t log(\frac{t_{t-1}}{\bar{t}}) + rho_{t,b} log(\frac{b_{t-1}}{\bar{b}}) + \epsilon_{t,t}. \tag{27}$$

The tax rule is amended by a term depicting the stance of the economy with $rho_{t,y} > 0$,

$$log(\frac{t_t}{\bar{t}}) = rho_t log(\frac{t_{t-1}}{\bar{t}}) + rho_{t,b} log(\frac{b_{t-1}}{\bar{b}}) + rho_{t,y} log(\frac{y_{t-1}}{\bar{y}}) + \epsilon_{t,t}. \tag{28}$$

The fiscal policy reacts in a countercyclical way to the stance of the economy in the previous period, i.e. there is fiscal support in the form of lower current taxes given the contraction of the economy in the Covid pandemic, for example. In consequence, less income is channelled through to the government in which case more debt is issued in turn with higher taxes to follow sequentially.[3]

2.6 Conventional Monetary Policy

Conventional monetary policy focuses on the savings rate as the policy rate which gets described by the standard Taylor Rule with intertia:

$$log(\frac{r_{s,t}}{\bar{r_s}}) = rho_s log(\frac{r_{s,t-1}}{\bar{r_s}}) + rho_y log(\frac{y_t}{\bar{y}}) + rho_\pi log(\frac{\pi_t}{\bar{\pi}}) + \epsilon_{r,s,t}, \tag{29}$$

$\bar{r_s}$, $\bar{y}$ and $\bar{\pi}$ are targets or steady state values and $\epsilon_{r,s,t}$ is a normal monetary shock with mean zero and standard deviation $\sigma_{r,s}$.

[3] A more elaborated fiscal sector would allow for varying the response and to differentiate in between taxes and expenditures with the impact on the economy, given that expenditures are merely an AR(1) process in the model.

2.7 Shocks

In general notation, the sequence of all exogenous variables $shk_t = \{shk_{s,t}, shk_{l,t}, shk_{z,t}, shk_{b,f,t}, shk_{g,t}\}$ takes the shape of AR(1) processes in log deviation from the steady state,

$$\log(shk_t) = (1 - \rho_{shk}) \log(s\bar{h}k) + \rho_{shk} \log(shk_{t-1}) + \epsilon_{shk,t}, \tag{30}$$

where $\epsilon_{shk,t} \approx N(0, \sigma^2_{shk})$ and $\rho_{shk} \in [0, 1)$.

All the error terms in the AR(1) processes are normally distributed with the mean zero and the variance σ^2 to be specified for each shock. Normally distributed disturbances are furthermore included in the processes for the standard Taylor Rule, the Taylor type policy rule for QE defined in the next section and the response of taxes in the fiscal policy rule, $\epsilon_t = \{\epsilon_{r,s,t}, \epsilon_{b,cb,t}, \epsilon_{t,t}\}$.

3 Quantitative Easing

3.1 Theoretical Aspects

The following considerations bridge the gap from several theoretic underpinnings not covered in the DSGE literature, which is summarized in the introduction, to the model formulation in section 3.2. Especially the intersection of fiscal with monetary policy revealed by the optimization problem of the central bank in the conduct of QE and the incentives problem arising out of QE for the conduct of fiscal policy are reasoned henceforth. Two financial channels in the model are related to QE with regard to financial stability.

Out of a long term perspective, Borio (2020) steadily argues that the costs of this form of monetary policy become yet more visible. In face of diminishing effectiveness of monetary policy with regards to the credit channel and diminishing responsiveness of inflation to monetary policy, it is essential over the longer term to rebuild the policy buffers depleted. Crucially, monetary policy itself has contributed to the higher stake at risk or the worse conditions of exiting that policy because it increased the sensitivity of participants to it. Naturally, participants have been adjusting to the very accommodative, ongoing stance of monetary policy and immense debt burdens have been created. It is stated explicitly the sensitivity of financial markets with the monetary policy under financial dominance but also under fiscal dominance. The evolution of the real debt levels of selected Eurozone members with the accompanied yields in Appendix A is indicative in the context of fiscal dominance. It is obvious yet critical that these missing efforts of debt consolidation occur even on top of the positive effect of QE on the economies of Southern Europe in terms of alleviating the costs on sovereign debt and the accompanied debt levels. In other words, a huge fraction of members of the Eurozone have continuously not adhered to the debt limit of the Maastricht Treaty which was posed with the aim to, amongst others, maintain monetary independence.

The decision pursue sovereign bond purchases in the Eurozone give rise to concerns, at the very minimum, about the independence of the central bank and the interference with fiscal policy.[4] Monetary and fiscal policy gets increasingly intertwined since policy makers rely on the central bank in that the higher debt burden does not become unsustainable as long as policy rates remain at low levels. A revaluation of the debt levels and fundamentals by the financial markets, such as in the sovereign debt crisis in the Eurozone in 2010/2011, brings about the opposite outcome given that the unsustainable public policies in several Eurozone countries lingered on. Accordingly, the assumption will naturally be tested first for the public entities at the brink of insolvency, these are at the extreme Greece and Italy. There might be multiple equilibria involved and the yields required on bonds might remain at a sustainable level for some debt level, beyond which they rise in an accelerating pace. The view of a possible, sudden rise in bond yields was shared by Borio already before the COVID-19 pandemic (Schrörs (2019)). By the third quarter of 2021, at the very latest, it became evident that inflation had to be persistently above target, along the rising, negative consequences of the spurred inflation on consumption, savings and output. Regarding public finances, the problems of exiting the ultra-loose stance of monetary policy become increasingly apparent. The question is raised what risk the policy of the ECB poses for the sustainability of debt finances if it is required to either decrease sovereign debt holdings or to increase the policy rate in order to fulfill its mandate of price stability. Overall, central banks are in general constraint in what they are capable to achieve, both economically and legally, and the overall European institutional framework is subject to the outcome of political voting.[5]

There are several distorted incentives for fiscal policy created by the announcement and conduct of government bond purchases. It is not only the moral hazard problem in effect ignored. In addition, the market mechanism is distorted by the suppression of bond yields, which do not mark anymore the level sustained by supply and demand. The market mechanism is essentially deprived of its information and coordination function. The freerider problem applied to fiscal policy as well as the externality of the public policy of a single member posed on other members of the Eurozone are attached to it. Apparently, a sovereign debt restructuring mechanism is still not considered by politicians whilst discussed for the Eurozone already in 2010 by Franz et al. (2010). The temporary exit of the member with unsustainable debt finances from the Eurozone, not the European Union, would provide the member with the possibility to devalue its currency and thereby align relative

[4]With regards to the statement of the ECB president Lagarde in March 2020 that it was not the job of the ECB to close spreads, a governing council member stated according to the Financial Times in March 2020: "She just lost concentration. It was her first major policy decision and it was a very complicated situation. I think her mind was somewhere else." (Arnold (2021))

[5]Havlik and Heinemann (2020) compare the PSPP and the PEPP and analyze the execution of the programs. Quote Havlik and Heinemann (2020, p.17): "In its PSPP ruling, the European Court of Justice (ECJ) had stressed the importance of constraining rules and the capital key orientation for its own Art. 123 compliance test. In its PSPP judgement from 11 December 2018 (C-493/17) that answered the German Federal Constitutional Court's request for a preliminary ruling, the ECJ discusses a possible infringement of Art. 123. Here, the ECJ explicitly acknowledges the argument that the ECB purchases bonds "in accordance with the key for the subscription of the ECB's capital" rather than "with other criteria such as, for example, the level of the respective debts of each Member State" (C-493/17, nb. 140). The Court further acknowledges that this safeguard avoids the risk that countries could provoke higher purchases of their debt with increasing public deficits." The extreme capital key deviations for Italy and Spain under the PSPP with its continuation under the PEPP are particularly striking.

prices vis-à-vis other economies. Additional arguments against quantitative easing are outlined by Borio (2014) and consistently by Borio (2020).

Finally, there are not negligible risks involved for financial stability if bubbles are created in the asset as well as housing markets, in line with the associated rents and mortgages to adjust to the higher level which reduces the disposable income of households. In turn, financial stability is of paramount importance for the overall economy as the financial crisis of 2007/08 in the USA with its spillover effects revealed. There are several channels through which the downturn in the financial cycle affects the real economy. Among those, the bank lending channel is essentially a supply side channel, which incorporates the capital ratio of the bank. The agency-cost models go further in that lending is determined endogenously by shifts in the value of the underlying assets with a feedback effect. A more common name of the feedback loop in the downturn of the economy, while comprising asset prices, collateral and the supply of credit, is the debt-deflation mechanism.

3.2 QE in the Model

The description of the savings rate required on deposits is provided by the Taylor Rule in equation (29) while the yield on long term government bonds is modelled differently. The two interest rates are targeted by different policy actions. This allows for a more realistic scenario amid the intersection of monetary and fiscal policy because the two interest rates might follow different paths, with the latter only associated with the public sector. When the ZLB is binding, it refers to the short term policy rate. Thus the no-arbitrage condition with respect to the different assets of deposits and government bonds would only be fulfilled in the model if the banks valued the assets equally. The purchase of long term government bonds marks the unconventional monetary policy of the ECB, not direct lending to the private sector encountered in most of the previously discussed literature. As such, the QE aimed at impacting the yield curve at the longer end is figured out accurately. On the balance sheet of the central bank, the funding of the bond purchases by reserves in not explicitly modelled, rather are the reserves assumed to evolve accordingly. The model will capture the incentives structure in part by hinting at a Moral Hazard problem involved. The term $+\chi_{qe1}(\frac{b_t}{b})^{\chi_{qe2}} b_{cb,t}$ in the condensed budget constraint of the government, equation (24), signals that the willingness of fiscal authorities to reduce the current debt is negatively related to the amount of sovereign bond purchases in the current period while the effect is highly sensitive to the parameters χ_{qe1} and χ_{qe2} in the functional form.

The goal of the central bank entering the regime of monetary financing is to maximize the seignorage under the budget constraint of the government, equation (24),

$$\max E_0 \sum_{t=0}^{T} \beta_{cb}^t d_{cb,t}, \tag{31}$$

with the seignorage

$$d_{cb,t} = p_{b,t-1} b_{cb,t-1} \frac{r_{b,t}}{\pi_t} - p_{b,t} b_{cb,t}. \tag{32}$$

The ansatz might vaguely be related to the accounting view of a consolidated balance sheet of the government and the central bank as presented by Niepelt (2020).[6] Alternatively, a specific utility function attached to the proceeds from sovereign bond purchases can be defined instead of the seignorage to be maximized. Obviously, the ansatz outlined is less qualified for members such as Germany with relatively low bond yields. The optimization of the central bank with respect to the quantity of government bond purchases yields the following expression:

$$0 = \beta_{cb} p_{b,t} E_t(\frac{r_{b,t+1}}{\pi_{t+1}}) - p_{b,t} + (\frac{1}{c_t^\tau})\chi_{qe1}(\frac{b_t}{\bar{b}})^{\chi_{qe2}} + \chi_{qe1}\chi_{qe2}(\frac{b_t}{\bar{b}})^{\chi_{qe2}-1}\frac{b_{cb,t}}{\bar{b}}, \tag{33}$$

β_{cb} denotes the time adjustment factor for the central bank regarding the budget constraint of the government. The equilibrium condition can be rearranged to result in the expression for the expected yield

$$\beta_{cb} p_{b,t} E_t(\frac{r_{b,t+1}}{\pi_{t+1}}) - p_{b,t} = -(\frac{1}{c_t^\tau})\chi_{qe1}(\frac{b_t}{\bar{b}})^{\chi_{qe2}} - \chi_{qe1}\chi_{qe2}(\frac{b_t}{\bar{b}})^{\chi_{qe2}-1}\frac{b_{cb,t}}{\bar{b}}. \tag{34}$$

It can be derived that contrary to the market based determination given by equation (15), the expected, discounted yield is a negative function of the derivations of the term of Moral Hazard which feature the deviation of the current level of public debt from its long-run trend and the current bond purchases by the central bank. Thereby, it is assured in the model that the expected yield is suppressed by QE.

The evolution of bond purchases decided upon by the central bank is modelled via a policy reaction function. Like in Cúrdia and Woodford (2011) and Sims and Wu (2019), there can be derived a Taylor-type rule for the increase in sovereign bond purchases depending on the deviations of inflation and output from their steady state levels, marked by $\rho_{\pi,cb}$ and $\rho_{y,cb}$, respectively. It thus defines an endogenous reaction function in addition to the standard Taylor Rule (29):[7]

$$log(\frac{b_{cb,t}}{\bar{b_{cb}}}) = rho_{b,cb} log(\frac{b_{cb,t-1}}{\bar{b_{cb}}}) - \rho_{y,cb} log(\frac{y_t}{\bar{y}}) - \rho_{\pi,cb} log(\frac{\pi_t}{\bar{\pi}}) + \epsilon_{b,cb,t}. \tag{35}$$

When examining the equilibrium effects in the financial sector, the balance sheet constraint of financial intermediaries is potentially relaxed by QE due to the lower expected yield and equivalently higher prices of bonds channelling through. For the financial intermediaries, government bonds con-

[6] The view of Niepelt (2020) however abstracts from the theory of incentives for the conduct of fiscal policy and the overall economic rationale for independent institutions. Likewise, the aggregate level of sovereign debt and the interest rate on it would not play any role. Moreover, it is the aggregated balance at any given point of time and as such it abstracts from dynamic decision making.

[7] The rule easily allows for extensions such as the indicator of the bond yield or the yield spread.

stitute an alternative asset class besides the credit exposure which in principle allows for a crowding out effect as in Van der Kwaak and Van Wijnbergen (2017).

The term structure in yields $sp_{r,t}$ is created similar to the one examined by Christiano et al. (2014), with the difference that the short term rate is marked by the savings rate instead of a short term bond,

$$sp_{r,t} = r_{b,t} - r_{s,t}. \tag{36}$$

Remarkably, it is not discussed the scope of monetary financing by the ECB, bearing in mind that the ECB has actually no mandate to finance governments. Rather, the sovereign debt purchases within the implementation of the prior ECB policy decisions are taken as given and are not discussed else than by the effects pinned down through the applied New Keynesian DSGE model. Apart from the economic perspective, the model does not dive into the political or legal aspects of the decisions.

3.3 The Dynamics of QE

For the purpose of illustration of QE in the model, the dynamics in the economy are discussed when QE is increased from the assumed steady state level of 0.1 for $b_{cb,t}$ by the magnitude of one standard deviation of the estimated shock size. The corresponding magnitude of the shock is an increase in government bond purchases of about 0.03 in levels or 3 percent of GDP, respectively. The coefficients surrounding the model equations of government bond purchases are determined by the data. The Appendix provides information on the Bayesian estimation as well as the complete series of IRFs. All responses are in levels and deviations from the steady state. The sequence of dynamics reveals the intended drop in the government bond yield when the stock of sovereign debt moves from the balance sheet of the financial intermediary to the one of the central bank:

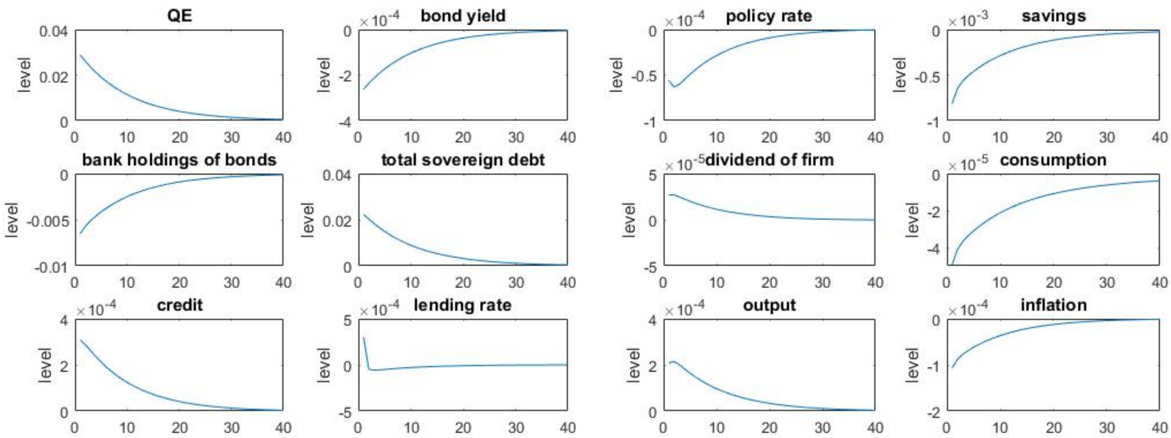

Figure 1: The dynamics of QE, selected IRFs.

The total sovereign debt does not remain constant but is fostered due to the wrong incentives associated with the monetary financing of the central bank. The crowding out effect on the balance sheet of the financial intermediary is affirmed by the lower bond exposure and higher level of credit in the economy. Due to higher labor, or less unemployment respectively, the output rises and the intermediate goods producing firm is more profitable which puts downward pressure on the expected lending rate through the bank lending channel. Notably, credit is spurred although the lending rate is initially at a higher level and although the policy rate is alleviated in response to the drop in inflation. Consumption and wages follow lower paths as does inflation. Along the lower exposure to sovereign debt, the financial intermediary resorts to less financing via deposits. In quantities, the installment of QE of around 3 percent of GDP has the effect of suppressing the bond yield by around 0.024 percent, of spurring public debt by around 1.7 percent, credit by around 0.03 percent, output by around 0.02 percent and decreasing inflation very mildly by around 0.01 percent.

4 ZLB

4.1 ZLB in the Model

The ZLB is implemented in Dynare/MATLAB via the OccBin Toolkit developed by Guerrieri and Iacoviello (2015) inspired by Uhlig (1995), cit. both in Holden (2016) and Sims and Wu (2019), and outlined in Canova (2017). Guerrieri and Iacoviello (2015) apply a piecewise linear solution method in the two regimes of binding and not binding ZLB, which "involves linking the first-order approximation of the model around the same point under each regime. Importantly, the solution that the algorithm produces is not just linear – with two different sets of coefficients depending on whether the occasionally binding constraint is binding or not – but rather, it can be highly nonlinear. The dynamics in one of the two regimes may crucially depend on how long one expects to be in that regime. In turn, how long one expects to be in that regime depends on the state vector. This interaction produces the high nonlinearity." (cit. Guerrieri and Iacoviello (2015, p.2)) The solution algorithm starts by guessing the periods under which each regime is active, followed by verifying or updating the guess.

Accordingly in the model, the ZLB refers to the deposit rate in gross terms which is the policy rate described by the standard Taylor rule with inertia. The occasionally binding constraint is defined by

$$r_{s,t} = \max(r_{s,t}, 1), \tag{37}$$

where the lower bound of 1 implies an interest rate of 0 percent. In the regime where the lower bound is reached, the model is around a constant $r_{s,t}$ of 1 whereas it behaves as before when the lower bound is not reached. An accompanying second occasionally binding constraint on the lending rate $r_{l,t}$ is imposed.

5 Parameter Values and Steady States

The parameters are partly estimated in Dynare via Bayesian estimation and partly calibrated. The estimation has the advantage that the parameters which are barely used in the literature, such as the elasticity parameters in the monitoring costs due to financial frictions, are in line both with the time series of the observables and the specific model. Likewise, the method of Bayesian estimation is preferable for the parameters estimating the effect out of quantitative easing to take an objective stance. The Appendix C summarizes data issues, data sources, as well as configurations and results of the Bayesian estimation.

5.1 Parameter Values

The parameters of the standard New Keynesian model β, τ, γ, δ, α, ρ_s, ρ_y and ρ_{pi} are reported in Pichler (2008) with reference to Ireland (2001). For the shock processes, a persistence of 0.9 is assumed. Moreover, the values for $\rho_{t,b}$ and $\rho_{t,y}$ in the fiscal rule are assumed. The estimated shock intensities are provided in Appendix C. The price adjustment parameter ϕ crucially enters the version of the New Keynesian Phillips curve in front of the inflation and the output gap and was subject to much dispute in the discussion about the persistent low HICP inflation before the COVID-19 pandemic. The statements range from complete invalidity of the Phillips Curve to validity but flattened. Differently to the value of 80 reported in Pichler (2008) and 84 in Alpanda et al. (2014), ϕ is set to 30 which is found by Angeloni and Faia (2010) to match the Calvo-Yun approach in the Rotemberg framework based on the frequency of price adjustment of four quarters. The source for θ is Pichler (2008), and Alpanda et al. (2014) with reference to Smets and Wouters (2007). χ_{s2} is used from Alpanda et al. (2014), for χ_{l2}, the authors refer to the value in Bernanke et al. (1999). For the capital adjustment parameter o, Alpanda et al. (2014) refer to Jerman and Quadrini (2012) for the dividend smoothing by financial intermediaries observed in the data. The value of ν is set to the requirement following the Basel III phase-in arrangements in BIS (2013). The parameters $\chi_{b1}, \chi_{b2}, \chi_{qe1}, \chi_{qe2}$ and $\rho_{\pi,cb}$ are estimated. Sims and Wu (2019) choose to mimic the response of conventional policy with the value for the parameter of endogenous QE conducted with respect to private bonds. Nevertheless, there is not any reference made to real data within the modeling of QE, neither does there exist any justification (a priori) for setting the policy parameter of unconventional policy to that value. The remaining parameters are calibrated to match the steady states values in the data. The time series range from Q1 2000 to Q4 2020. The negative coefficient for $\kappa = -0.0016$ deserves special attention because it implies a negative, if minuscule, consumption of equity capital by financial intermediaries. It states equivalently that in equilibrium, the financial intermediaries are even building up capital to finance their profitable business. The parameter $\rho_{p,b}$ is computed from the formula for the duration (average maturity) n in Van der Kwaak and Van Wijnbergen (2017): $n = 1/(1 - \beta\rho_{p,b})$ where n is the average maturity of 28 quarters of the German bonds purchased by the ECB under the PSPP. That is, $\rho_{p,b} = 0.9740$. The constant coupon payment of 0.0246 are computed from the series on the bond yield. In sequence, $p_{b,t}$ is computed from the bond pricing formula and the known steady state values of $r_{b,t}$, the coupon payment and $\rho_{p,b}$.

Table 1: Parameter Values

$PARAMETER$	$VALUE$
γ	10
θ	6
β	0.99
β_e	0.9956
β_{cb}	0.8691
χ_h	9.7339
δ	0.025
α	0.36
ϕ	30
ν	0.105
τ	2
κ	-0.0016
σ	0.001
$\rho_{shk,s}$	0.9
$\rho_{shk,l}$	0.9
$\rho_{shk,z}$	0.9
$\rho_{shk,b,f}$	0.9
ρ_t	0.9
ρ_g	0.979
ρ_s	0.7
ρ_π	1.5
ρ_y	0.5
$\rho_{y,cb}$	0
$\rho_{\pi,cb}$	6.566
$\rho_{t,y}$	1.1
$\rho_{t,b}$	0.1
o	50

(Continued on next page)

Table 1: (continued)

$PARAMETER$	$VALUE$
$\rho_{p,b}$	0.9740
$coupon$	0.0246
χ_{s1}	0.9884
χ_{s2}	0.002
χ_{l1}	0.8752
χ_{l2}	0.05
χ_{qe1}	0.009
χ_{qe2}	0.115
$\rho_{b,bc}$	0.9
χ_{b1}	0.989
χ_{b2}	0.067

5.2 Steady States

The steady states of all variables implied by the calibrated and estimated parameters are summarized in the following table.

Table 2: Steady States

$VARIABLE$	$STEADYSTATE$
sp_r	0.0287
tr	-0.1806
t	0.2590
b_{cb}	0.1005
b_f	0.8709
r_b	1.0289
p_b	0.4479

(Continued on next page)

Table 2: (continued)

$VARIABLE$	$STEADYSTATE$
b	0.9714
g	0.4300
w	1.7828
c	0.4280
r_k	0.0351
k	8.5822
h	0.3004
s	1.1893
y	1.0042
x	0.2146
l	0.9952
k_f	0.1959
pi	1.0018
d_e	0.1630
d_f	0.0706
r_l	1.0063
r_s	1.0002
$d_{k,f}$	-0.0003

6 Policy Analysis

6.1 Productivity Shock

The policy analysis commences with a positive productivity shock to the economy where the shock intensity is such that the policy rate hits the ZLB. The blue, solid line marks the case of the binding ZLB. The dashed, red line evaluates the model when there is not restriction on the policy rate.

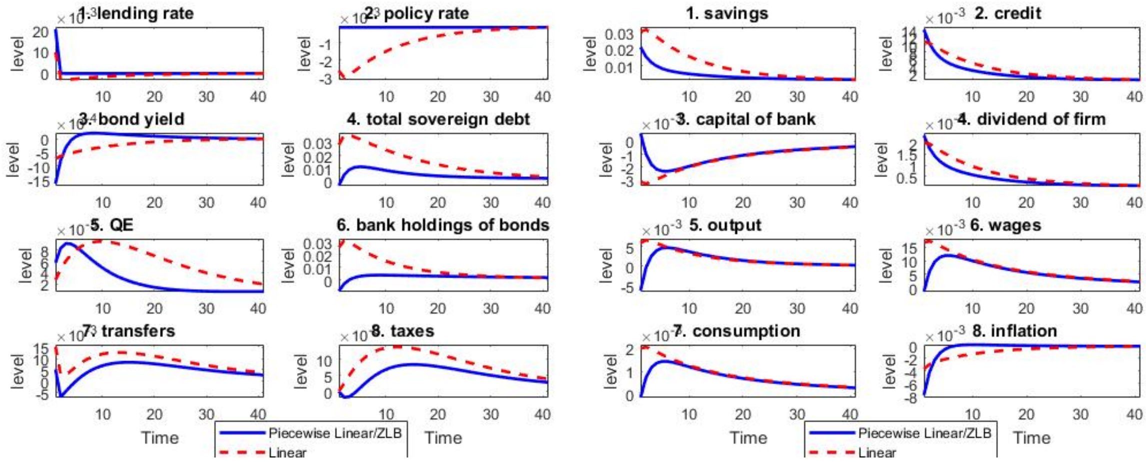

Figure 2: Productivity Shock, selected IRFs.

When there is no floor on the savings rate, the labor augmenting productivity shock increases overall output, consumption and wages as the central bank is capable to provide the necessary accommodation. The effects and the accommodative stance of monetary policy are yet not sufficient to lead to a higher path of inflation. Longer lasting dividends are accrued by the intermediate goods producing firm and more credit is demanded. The banking sector expands its balance sheet to provide more credit and more sovereign debt is purchased. Looking at the financing side, the financial intermediaries are in total capable to cut on its position of equity capital given the considerably higher accumulation of deposits. If the constraint is binding, in comparison, monetary policy cannot response to the subdued inflation via the policy rate and QE is initially conducted more in a reaction to the lower inflation. The bond yield is suppressed by QE, yet are the related effects on the total level of public debt of minor quantitative extent in the model estimated on data for Germany. At the same time, inflation returns to its steady state more quickly. At the balance sheet of the banks, the lower paths of deposits and credit are related to the lower path of bank holdings of sovereign debt. Output, wages, consumption and inflation all reveal lower responses to the shock in the first periods given that the two nominal interest rates have no leeway to adjust downwards which would push down accordingly the real interest rates.

6.2 Monetary Shock

The scenario of a monetary shock produces the following IRFs:

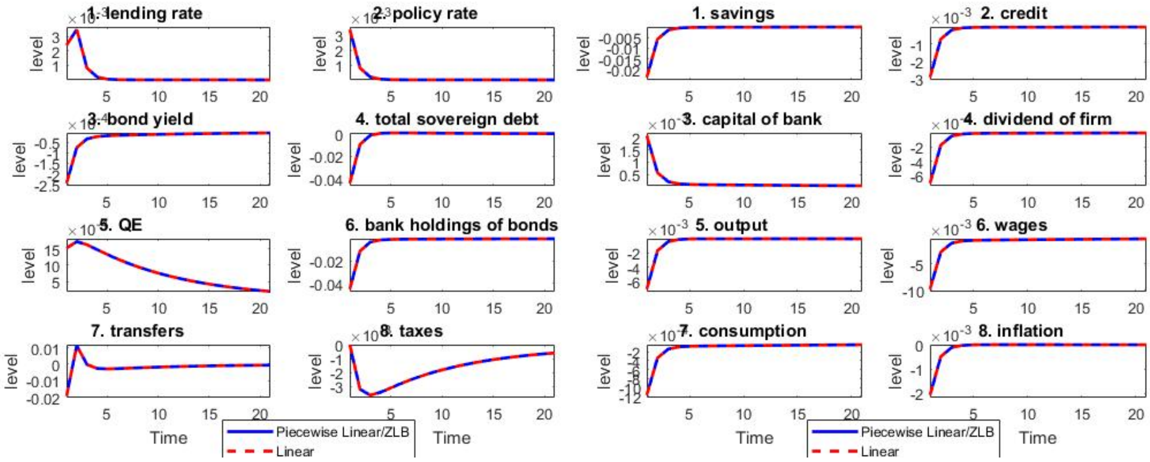

Figure 3: Monetary Shock, selected IRFs.

The monetary shock drives up the policy rate and due to the price setting of financial intermediaries the lending rate with a lag. Accordingly, the demand for deposits and credit falls and along the deleveraging process, the financial intermediaries cut on the quantity of sovereign bonds. To a similar extent, the total amount of outstanding total sovereign debt is reduced as QE is only activated by roughly 0.0017 in level terms. In the real economy, the dividends of the firm fall due to the lack of available credit and the intended drop in inflation occurs with the drop in output, wages and consumption. Like in the standard New Keynesian Model, the household faces higher utility out of future consumption in comparison to current consumption due to the higher real interest rate and, hence, postpones consumption to the future. Through the corresponding optimal decision regarding labor versus leisure, wages fall with consumption.

6.3 Shock to the Credit Supply

The next scenario is an orthogonal shock to the standard deviation within the AR(1) process fueling the monitoring costs of financial intermediaries with respect to the provision of credit to the intermediate goods producer. A shock to the net worth of a firm in a pandemic can be subsumed under this scenario. Moreover, the channel appears in the downturn phase in the financial cycle, in which the financial intermediary cuts back on credit. The shock is of such an intensity that both constraints are binding. Notably, a device in the Occbin toolbox with two binding constraints is that the second constraint, i.e. on the lending rate, is potentially binding only if the first constraint on the policy rate is binding.

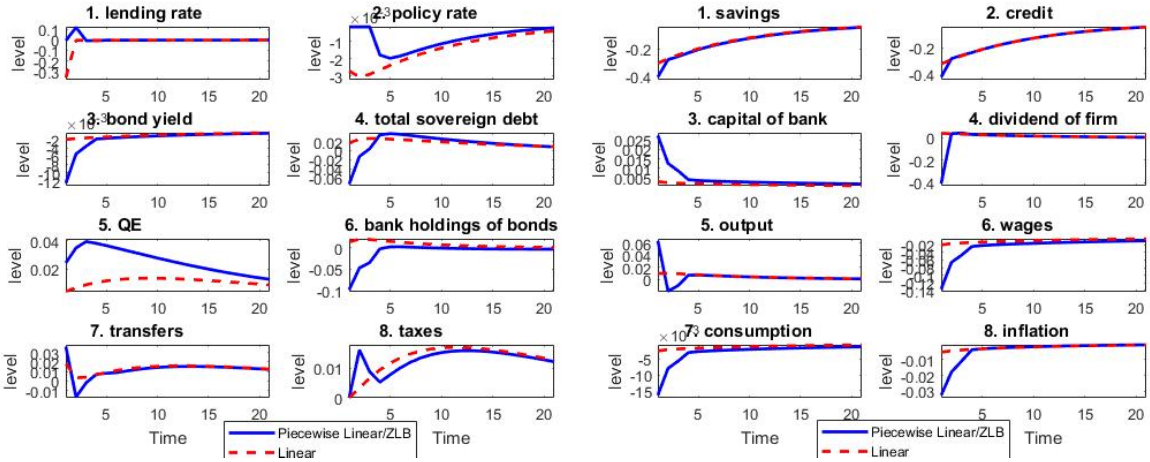

Figure 4: Shock to Credit, selected IRFs.

Without the ZLB binding, the higher monitoring costs for financial intermediaries increase the lending rate only in expectation while the actual lending rate is cut following the accommodation of monetary policy by lowering the policy rate and the huge drop in credit. The deleveraging process on the financial side is particularly obvious when looking at the very similar magnitudes of the reduction of savings and credit. Without any bound on the lending rate, the firms react to the drag on credit by accruing more profits and distributing dividends to alleviate the costs for banks in the bank lending channel to take on more loans. In the constrained case, it is interesting to observe that the lending rate even raises for one quarter as an response to the shock because the policy rate remains very close to its initial level. Since a variation of the lending rate is not available to spur credit growth, the drop in credit is more pronounced for the initial period. Accordingly, savings drop more with the higher level of the savings rate. Moreover, when the lower bound on the policy rate is in place, the difference is that QE is activated which pushes down the yield on long term bonds in the first periods. In the deleveraging process, financial intermediaries cut on the exposure of sovereign debt and, very much in line with it, overall sovereign debt falls. Considerably more equity is raised. In reaction to the more pronounced deflation, the conduct of QE and the accompanied drop in the long term bond yield is not sufficient in magnitude to prevent the fall in the public debt level. This comes to no surprise when looking at the magnitudes of higher QE by around 0.007 in period 2 while the deleveraging amounts to more than 0.4 in level terms, which corresponds to around 40 percent of all the credit available to entrepreneurs. When examining the budget of the sovereign, the transfers to the household are initially at a higher level following the lower overall level of public debt. Due to the fiscal rule imposed, higher taxes are levied on the household in a response to the initially higher stance of output. If monetary policy is restricted by the ZLB, it results in a initally higher but very quickly contracting economic activity and wages, consumption and inflation drop considerably more.

6.4 Shock to Public Debt Held by the Financial Intermediary

In sequence, the scenario is presented in which a shock hits the financial intermediary fueling the monitoring costs with respect to its exposure to sovereign debt. Subsumable are risk shocks to the debt level, a change in investor sentiment and variations around the public debt, which might for instance be the case if investors evaluate fundamentals in the economy differently and project it to the public debt level. Due to the market mechanism, the expected and, as visible in the IRFs, also actual bond yield are pushed up when the lower bounds are not in place and QE is almost not initialized at all, and the banks holdings of sovereign debt drop sharply. The shock intensity is such that the economy is driven to the zero lower bound:

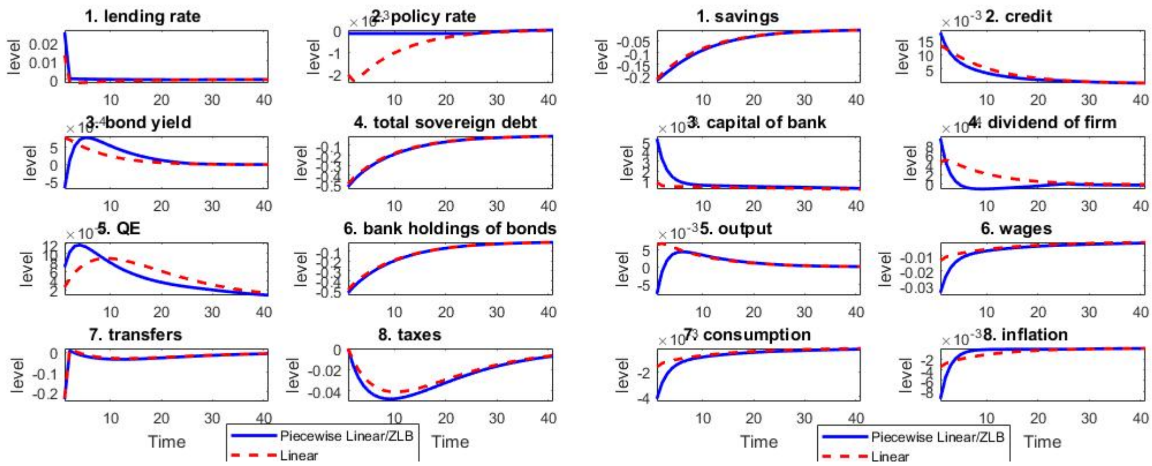

Figure 5: Shock to Public Debt Held by the Financial Intermediary, selected IRFs.

Independent of the two lower bounds, the overall sovereign debt drops in line with the sell off which occurs in the financial sector, consequently it is cut on savings in financing. The public entity manages to sequentially cut taxes whilst the public debt is reimbursed by the initial drop in transfers to the households. The transfers are quickly resumed to return to the steady state in about two periods, after which they are again decreased. There are several distinct developments to report for the occasionally binding constraints in place. First of all, like in the previous scenario of a shock to the credit to the firm, QE is activated if there is no more leeway on the policy rate and the bond yield is initially lowered by the activation of unconventional monetary policy. The installment of QE amounts to roughly 0.12 in level terms after two quarters. The lower bound on the policy rate is accompanied by the higher level of the lending rate, and the stance of credit is higher for about one period due to the higher dividends accrued which drives down the monitoring costs for banks. As bond holdings are gauged a little bit lower with the conduct of QE, an accompanying reason for the higher stance of credit constitutes the crowding out effect to occur on the balance sheet of the financial intermediary.

Accordingly, equity capital is raised considerably more and output is considerably lower in the case of the binding constraints on the interest rates. The dynamics of the drop in the wage level in the economy, as well as the drop in both the consumption and the inflation all reveal the depressing effect of the higher monitoring costs for the financial intermediary. Furthermore, all the three paths are gauged lower if the ZLB constraints monetary policy.

7 Conclusion

In the model fed with the relevant time series data for Germany, QE has the announced effect regarding the drop in the government bond yield at the expense of a rise in public debt. The model dynamics display the intended crowding out effect on the balance sheet of the bank spurring credit and output. Amid lower levels of wages and consumption, the overall quantitative effect is nevertheless not inflationary but deflationary.

When the economy faces a positive productivity shock, the two lower bounds on the banking interest rates lead to a considerably weaker performance of the real economy and a deleveraging process on the financial side, which is in magnitude not sufficiently counteracted by the initialization of QE. Nevertheless, the steady state level of inflation is more rapidly reached with the initialization of QE. After a shock to the credit supply, QE is again activated more if the ZLB on the policy rate is in place and, hence, the yield on long term bonds is tilted downwards. The deleveraging process on the financial side is of such an extent that overall public debt falls with the reduction of its exposure at banks. Output after the first period, consumption, wages and inflation drop more in the case of the ZLB whilst QE does not make up for the loss. The same findings for the economic performance marked by these four variables are obtained for the analysis at the ZLB when a shock hits the exposure of public debt at banks. Likewise, the conduct of QE is enhanced with the desired effect of initially pushing down the yield whereas the estimated effect related to the distorted incentives driving up the public debt does not play a role in terms of changing significantly the outcome induced by the decision of banks. Furthermore, QE in the latter scenario leads to a relatively small crowding out effect on the balance sheet of the bank.

There are several robustness and sensitivity analyses to be conducted: in addition to the sensitivity analyses related to the parameter values of the New Keynesian Phillips Curve and the coefficients attached to monetary and fiscal policies, the overall model can be easily compared against established models following a monetary policy shock. Alternatively can a single model be studied under different policy rules as described in Wieland et al. (2016). Amongst the various, potential extensions of the model, the most promising and straightforward one is an elaboration of the fiscal sector with various fiscal instruments to study the fiscal policies in depth. In order to control the public debt level, a budget deficit rule and a fiscal rule balancing total public debt are activated. That further allows to explore the core of the model, that is the intersection of monetary with fiscal policy. A reference is made to the relatively rich elaboration of the fiscal sector in Stähler and Thomas (2011) cit. in publication of (2012) in Cavalcanti and Vereda (2015) and Cavalcanti et al. (2018). Additional extensions constitute the analysis and the inclusion of broad money supply with its effect on inflation to define an indicator for it, as well as to account for the demographic change in Germany with reference to Goodhart and Pradhan (2021). Finally, further research can proceed along the dimensions of estimating a greater amount of parameters and the application of numerical methods for the critical parameters.

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Appendix A: Debt-to-GDP ratios and 10 year government bond yields in percent for selected Eurozone members

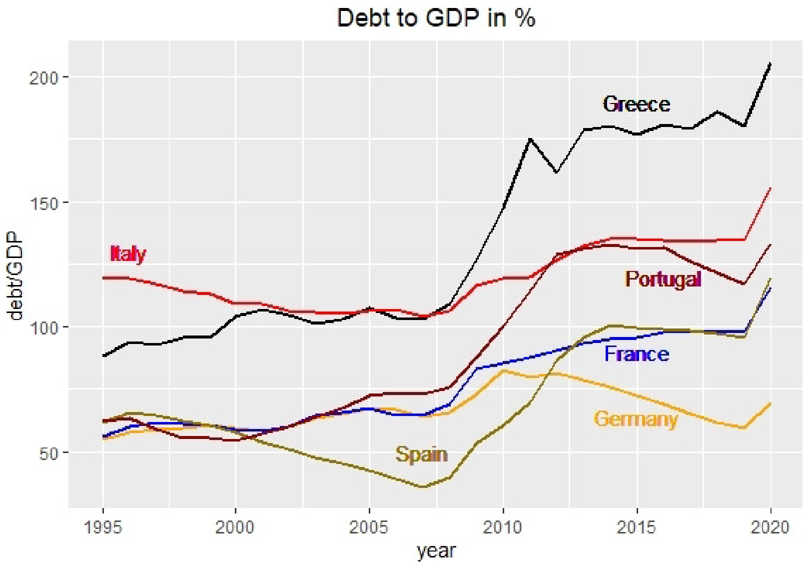

Figure 6: Debt-to-GDP ratios in percent for selected Eurozone members. Source: ECB (2021).

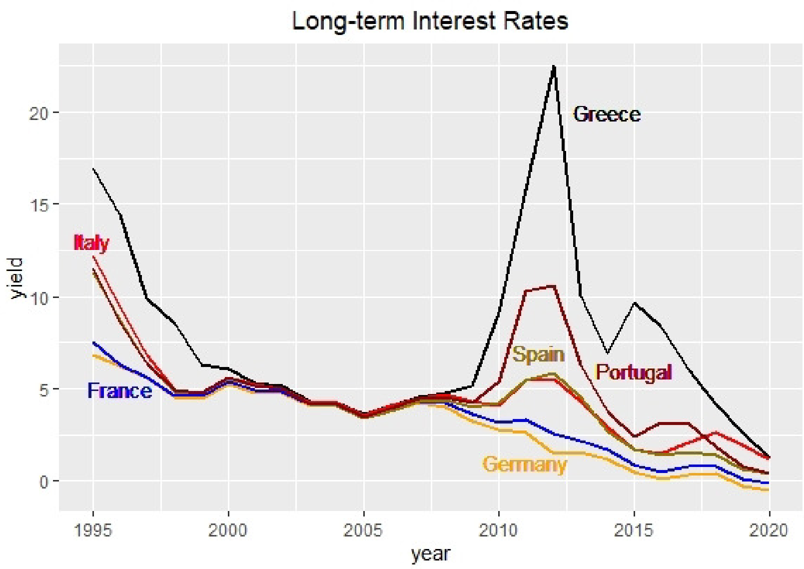

Figure 7: 10 year government bond yields in percent for selected Eurozone members. Source: ECB (2021).

Appendix C: Optimization problems and equilibrium conditions

Household

The representative household maximizes expected utility in equation (1) under the budget constraint (2):

$$\max U = E_0 \sum_{t=0}^{T} \beta^t \{[c_t^{1-\tau} + \chi_h(1-h_t)] + \lambda_t [w_t h_t + r_{k,t} k_t + \frac{r_{s,t-1} s_{t-1}}{\pi_t} + tr_t + d_{f,t} + d_{e,t} - t_t - c_t - x_t - q_{k,t} - s_t(1+\mu)_t]\} \tag{38}$$

The optimization with respect to c_t yields the expression for the Lagrangean multiplier λ_t:

$$\lambda_t = \frac{1}{c_t^\tau} \tag{39}$$

The remaining control variables $h_t, s_t, k_{t+1} and k_{f,t}$ give rise to the following equilibrium conditions:

$$0 = -\chi_h + \frac{w_t}{c_t^\tau} \tag{40}$$

$$0 = \frac{1}{c_t^\tau} \chi_{s1} [\nu \frac{l_t + b_{f,t}}{k_{f,t}}]^{\chi_{s2}} shk_{s,t} - \beta E_t \frac{r_{s,t}}{c_{t+1}^\tau \pi_{t+1}} \tag{41}$$

$$0 = \frac{1}{c_t^\tau}[1 + \gamma(\frac{x_t}{k_t} - \delta)] - \beta E_t \frac{1}{c_{t+1}^\tau}\{(r_{k,t+1} + 1 - \delta - (\frac{\gamma}{2})[(\frac{x_{t+1}}{k_{t+1}} - \delta)^2] + \gamma(\frac{x_{t+1}}{k_{t+1}} - \delta)(\frac{x_{t+1}}{k_{t+1}} - \delta + 1))\} \tag{42}$$

The budget constraint (2) with the expressions for the dividends of firms (7) and banks (13) inserted:

$$\begin{aligned} 0 = {} & y_t + tr_t - t_t + p_{b,t-1} b_{f,t-1} \frac{r_{b,t}}{\pi_t} - p_{b,t} b_{f,t} \{shk_{b,f} \chi_{b1} [(\frac{b_t}{\bar{b}})^{chi_b 2}]\} - \frac{\phi}{2}\{[(\frac{\pi_t}{\bar{\pi}}) - 1]^2\} y_t + \\ & l_t - l_t(1+\zeta)_t + s_t - s_t(1+\mu)_t - c_t - x_t - (\frac{\gamma}{2})[(\frac{x_t}{k_t} - \delta)^2] k_t - (\frac{\o}{2})[(\frac{d_{k,f,t}}{k_{f,t}} - \kappa)^2] k_{f,t} - d_{k,f,t} \end{aligned} \tag{43}$$

The law of motion for physical capital:

$$0 = k_{t+1} - (1-\delta) k_t - x_t \tag{44}$$

Firms

Final goods producer

The objective function of the final goods producer under perfect competition with the constant returns to scale production function in equation (5):

$$\max_{y_{j,t}}[y_t p_t - \int_0^1 y_{j,t} p_{j,t}\, dj] = \max_{y_{j,t}}[(\int_0^1 y_{j,t}^{\frac{\theta-1}{\theta}}\, dj)^{\frac{\theta}{\theta-1}} p_t - \int_0^1 y_{j,t} p_{j,t}\, dj] \tag{45}$$

The optimality condition for the input $y_{j,t}$ yields:

$$y_{j,t} = (\frac{p_{j,t}}{p_t})^{-\theta} y_t \tag{46}$$

Accounting for the optimal input $y_{j,t}$, the objective function becomes:

$$\max_{y_t}[y_t p_t - \int_0^1 (\frac{p_{j,t}}{p_t})^{-\theta} y_t p_{j,t}\, dj] \tag{47}$$

The optimization with respect to final output y_t yields the expression for the price of the final good p_t

$$p_t = (\int_0^1 p_{j,t}^{1-\theta}\, dj)^{\frac{1}{1-\theta}} \tag{48}$$

Intermediate goods producer

The optimization problem of the intermediate goods producer under monopolistic competition is to maximize the real dividends as described by (6) under the constraint of the production technology (8), and given the optimal demand for inputs by final goods producer from (46):

$$\max d_{e,t} = E_0 \sum_{t=0}^{T} \beta_e^t \{\lambda_t[\frac{p_{j,t}}{p_t}^{1-\theta} y_t + l_t - r_{k,t}k_{j,t} - w_t h_{j,t} - \frac{r_{l,t} l_{t-1}}{\pi_t} - q_{p,t}] + \omega_t[k_{j,t}^{\alpha}(shk_{z,t} h_{j,t})^{1-\alpha} - \frac{p_{j,t}}{p_t}^{1-\theta} y_t]\} \tag{49}$$

where ω_t is the Lagrangian multiplier for the production constraint.

The sequences of control variables $h_{j,t}, k_{j,t}, l_t, p_{j,t}$ fulfil the following equilibrium conditions, where $h_{j,t} = h_t$, $k_{j,t} = k_t$ and $p_{j,t} = p_t$ and the first equilibrium condition is derived from the combination of optimal h_t and k_t:

$$0 = \alpha w_t h_t - (1-\alpha) r_{k,t} k_t \tag{50}$$

$$0 = \frac{1}{c_t^{\tau}} - \beta_e E_t \frac{r_{l,t+1}}{c_{t+1}^{\tau} \pi_{t+1}} \tag{51}$$

$$0 = \frac{1}{c_t^{\tau}}[1-\theta+\theta\frac{w_t h_t}{(1-\alpha)y_t} - \phi(\frac{\pi_t}{\bar{\pi}}-1)\frac{\pi_t}{\bar{\pi}}] + \beta_e E_t[\frac{1}{c_{t+1}^{\tau}}(\frac{\pi_{t+1}}{\bar{\pi}}-1)\phi\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{\bar{y}}] \tag{52}$$

The cash flow:

$$0 = y_t + l_t - r_{k,t}k_{j,t} - w_t h_{j,t} - \frac{r_{l,t}l_{t-1}}{\pi_t} - q_{p,t} - d_{e,t} \tag{53}$$

The production technology constraint:

$$0 = k_t^{\alpha}(shk_{z,t}h_t)^{1-\alpha} - y_t \tag{54}$$

Financial intermediary

The financial intermediaries maximize the discounted value of expected dividends according to equation 11 under the balance sheet constraint 12. The Lagrangean reads as follows:

$$\max d_{f,t} = E_0 \sum_{t=0}^{T} \beta^t \{\lambda_t[\frac{r_{l,t}l_{t-1}}{pi_t} + \frac{r_{b,t}}{pi_t}p_{b,t-1}b_{f,t-1} + s_t - \frac{r_{s,t-1}s_{t-1}}{pi_t} - l_t(1+\zeta)_t - (1+\xi)_t p_{b,t}b_{f,t} - q_{k,f,t} - d_{k,f,t}] + \psi_t[p_{b,t}b_{f,t} + l_t - s_t - k_{f,t}]\} \tag{55}$$

where ψ_t is the Lagrangian multiplier associated with the production constraint, $(1+\zeta)_t$ defined by equation (18) and $(1+\xi)_t$ by equation (14), and $q_{k,f,t}$ by equation (17).

The sequential equilibrium conditions for $\{s_t, l_t, k_{f,t}, b_{f,t}\}$ hold in equilibrium. $\psi_t = \lambda_t – \lambda_t(1+\mu)_t = (1/c_t^{tau}) – (1/c_t^{tau})(1+\mu)_t$ arises from the optimal quantity of savings demanded by the financial intermediary and supplied by the household noted down in equation (41). The remaining conditions are:

$$0 = \beta E_t \frac{r_{l,t+1}}{c_{t+1}^{\tau}\pi_{t+1}} - (\frac{1}{c_t^{\tau}})[\chi_{l2}(1+\zeta)_t + (1+\zeta)_t] + (\frac{1}{c_t^{\tau}}) - (\frac{1}{c_t^{\tau}})(1+\mu)_t \tag{56}$$

$$0 = (\frac{1}{c_t^{\tau}})[1-\kappa-(\frac{ø}{2})(\frac{k_{f,t+1}}{k_{f,t}}-1)^2+ø(\frac{k_{f,t+1}}{k_{f,t}}-1)(\frac{k_{f,t+1}}{k_{f,t}})]+\beta E_t(\frac{1}{c_{t+1}^{\tau}})[-1-ø(\frac{k_{f,t+1}}{k_{f,t}}-1)]-(\frac{1}{c_t^{\tau}})+(\frac{1}{c_t^{\tau}})(1+\mu)_t \tag{57}$$

$$0 = \beta E_t \frac{r_{b,t+1}}{c^{\tau}_{t+1}\pi_{t+1}} - \frac{1}{c^{\tau}_t}[(1+\xi)_t + \chi_{b2}\chi_{b1}(\frac{b_t}{\bar{b}})^{\chi_{b2}-1} shk_{b,f,t}(\frac{b_{f,t}}{\bar{b}})] + (\frac{1}{c^{\tau}_t}) - (\frac{1}{c^{\tau}_t})(1+\mu)_t \tag{58}$$

The cash flow:

$$0 = \frac{r_{l,t}l_{t-1}}{\pi_t} + \frac{r_{b,t}}{\pi_t}p_{b,t-1}b_{f,t-1} + s_t - \frac{r_{s,t-1}s_{t-1}}{\pi_t} - l_t(1+\zeta)_t - (1+\xi)_t p_{b,t}b_{f,t} - q_{k,f,t} - d_{k,f,t} \tag{59}$$

The balance sheet constraint:

$$0 = p_{b,t}b_{f,t} + l_t - s_t - k_{f,t} \tag{60}$$

The law of motion for equity capital:

$$0 = (1-\kappa)k_{f,t} + d_{k,f,t} - k_{f,t+1} \tag{61}$$

Public sector / central bank

Public sector

The balance sheet:

$$0 = p_{b,t}b_{f,t} + t_t - g_t - tr_t - p_{b,t-1}b_{f,t-1}\frac{r_{b,t}}{\pi_t} + \chi_{qe1}(\frac{b_t}{\bar{b}})^{\chi_{qe2}} b_{cb,t} \tag{62}$$

The autoregressive process for public consumption:

$$\log(g_t) = (1-\rho_g)\log(\bar{g}) + \rho_{shk}\log(g_{t-1}) + \epsilon_{g,t} \tag{63}$$

The tax rule:

$$0 = rho_t log(\frac{t_{t-1}}{\bar{t}}) + rho_{t,b} log(\frac{b_{t-1}}{\bar{b}}) + rho_{t,y} log(\frac{y_{t-1}}{\bar{y}}) + epsilon_{t,t} - log(\frac{t_t}{\bar{t}}) \tag{64}$$

The bond pricing formula:

$$0 = \frac{coupon + \rho_{p,b}p_{b,t}}{p_{b,t}} - r_{b,t} \tag{65}$$

The total amount of sovereign bonds:

$$0 = b_{f,t} + b_{cb,t} - b_t \tag{66}$$

The yield spread:

$$sp_{r,t} = r_{b,t} - r_{s,t} \tag{67}$$

Central bank

The standard Taylor Rule with intertia:

$$0 = rho_s log(\frac{r_{s,t-1}}{\bar{r_s}}) + rho_y log(\frac{y_t}{\bar{y}}) + rho_\pi log(\frac{\pi_t}{\bar{\pi}}) + \epsilon_{r,s,t} - log(\frac{r_{s,t}}{\bar{r_s}}) \tag{68}$$

Under the regime of monetary financing, the objective of the central bank is to maximize the seignorage in equation (32) under the budget constraint of the government (24). The optimization problem reads as follows:

$$\max d_{cb,t} = E_0 \sum_{t=0}^{T} \beta_{cb}^t \{[p_{b,t-1} b_{cb,t-1} \frac{r_{b,t}}{\pi_t} - p_{b,t} b_{cb,t}] + \lambda_t [p_{b,t} b_{f,t} + t_t - g_t - tr_t - p_{b,t-1} b_{f,t-1} \frac{r_{b,t}}{\pi_t} + \chi_{qe1} (\frac{b_t}{\bar{b}})^{\chi_{qe2}} b_{cb,t}]\} \tag{69}$$

The optimization of the central bank with respect to $b_{cb,t}$ yields the equilibrium condition associated with QE:

$$0 = \beta_{cb} p_{b,t} E_t(\frac{r_{b,t+1}}{\pi_{t+1}}) - p_{b,t} + (\frac{1}{c_t^\tau}) \chi_{qe1} (\frac{b_t}{\bar{b}})^{\chi_{qe2}} + \chi_{qe1} \chi_{qe2} (\frac{b_t}{\bar{b}})^{\chi_{qe2}-1} \frac{b_{cb,t}}{\bar{b}} \tag{70}$$

The endogenous, Taylor-type reaction function for QE:

$$0 = rho_{b,cb} log(\frac{b_{cb,t-1}}{\bar{b_{cb}}}) - \rho_{y,cb} log(\frac{y_t}{\bar{y}}) - \rho_{\pi,cb} log(\frac{\pi_t}{\bar{\pi}}) + \epsilon_{b,cb,t} - log(\frac{b_{cb,t}}{\bar{b_{cb}}}) \tag{71}$$

Appendix C: Calibration and Estimation

Construction of the steady state value of credit to intermediate goods producer

The notation is as appears for the data series in the BIS Statistics Explorer:

F2.3: Total credit to the private non financial sector (core debt) in domestic currency billions
F2.6: Bank credit to the private non financial sector (core debt) in domestic currency billions
F3.3: Total credit to households (core debt) in domestic currency billions
F4.3: Total credit to non financial corporations (core debt) in domestic currency billions
where F2.3 = F3.3 + F4.3.

$\bar{l}$ = F4.3 x F2.6/F2.3 x F4.3/F2.3

The weight F2.6/F2.3 gives the relation of the bank credit versus the total credit to the corporations and households. The weight F4.3/F2.3 is constructed to account for the share of credit to corporations within the total credit to the private non-financial sector.

Data issues for calibration

In order to bring the savings rate closer to the short term policy rate and in face of data availability, the interest rate is confined to the rate on overnight deposits from households. Since total deposits at German banks are accrued for different and longer maturities agreed, the actual savings rate, which would result if weighted by all the different maturities, is presumably at a higher level yet follows a very similar path. Output is measured as the expenditure on final goods and services minus imports. The data on public spending and taxes is adjusted to a minor degree to account for the ratio of long term to total public debt. Hence, the simplifying assumption is made that government spending and taxes are equally distributed across the different maturities of public debt. For the use of pricing the long term sovereign debt, the bond yield is only provided for securities with a maturity of 10 years at the ECB Statistical Data Warehouse. With regards to the PSPP of QE, the cumulative net purchases of German public debt at Q1 2021 have the maturity of 6.7 years with the eligible maturity of 7.6 years at that point of time. The different maturities are long term debt securities from 1 until 31 years in the implementation of the program. In sum, the maturity of bonds purchased under the PSPP is roughly consistent with the data on the public debt used as $n = 7$ years for the bonds purchased, yet is the yield only available for 10 years. The assumed steady state of central bank purchases is obviously controversial. From a normative point of view the equilibrium is 0, yet is the relatively small quantity of 0.1 used due to the programs installed.

Bayesian estimation

Bayesian estimation is conducted with the computational software Dynare with help of Mancini Griffoli (2013), Pfeifer (2015) and Pfeifer (2020). There are 6 observables used for estimation. The data is retrieved from the time series on sovereign bond purchases by the ECB during the PSPP and PEPP programs, on credit as computed in Appendix C, total sovereign debt, inflation, consumption and the monetary policy rate. The time series are from Q1 2000 to Q4 2020 summing up to 84 observations for each variable. In comparison to the PSPP program, the minimum maturity of public purchases under the PEPP programme is shortened to 70 days, which decreases the average maturity of purchases to below 7 years. In fact, the average maturity of public securities from May 2020 to November 2020 is reduced to 4.115 years. Due to the lack of data which is disentangled for the different maturities, the aggregate measure is evaluated to account for the further considerable pilling up of public debt purchases under the PEPP programme. In order to remove seasonality from the time series of inflation, the MA(4) is constructed and the only measurement error included is on inflation $\epsilon_{pi,ME,t}$. In order to check whether stochastic singularity arises in the estimation, the identification power of the estimated parameters is evaluated. In total, there are 7 shocks estimated for 6 observables and any combination of two shocks affect some moments of the data of the observables differently. The shock process for taxes in the fiscal rule does not posses any identification power given the observables evaluated. In order to tilt the series on QE more towards stationary, first differences are used. Data on sovereign debt, credit and consumption are given in ratios to GDP. The observation equations are thus:

$$b_{cb,obs,t} = b_{cb,t} - b_{cb,t-1} \tag{72}$$

$$l_{obs,t} = l_t \tag{73}$$

$$b_{obs,t} = b_t \tag{74}$$

$$\pi_{obs,t} = pi_t + \epsilon_{pi,ME,t} \tag{75}$$

$$c_{obs,t} = c_t \tag{76}$$

$$r_{s,obs,t} = r_{s,t} \tag{77}$$

The Metropolis-Hasting algorithm is run on 4 parallel blocks and the scale for the jumping distribution is 0.7, which results in acceptance rates of 0.24, 0.26, 0.23 and 0.24, all numbers rounded to two decimals. The multivariate diagnostics reveals clear convergence for all the three aggregate measures of the 80 percent confidence interval around the mean, the measure of the variance and the third moments. The mode checks reveal that the modes of all parameters are at the maxima of the posterior modes and there are not any Blanchard Kahn conditions violated around the parameters spaces displayed. Among the experiments conducted was whether to include in the estimation the shock in the Taylor rule. The difference out of the Kalman filter in between the smoothed evolution of the measurement error on inflation and the monetary disturbance is revealing:

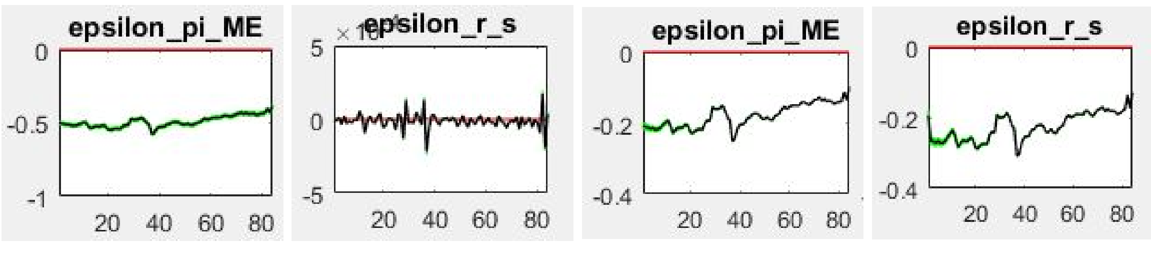

Figure 8: Comparison of the smoothed measurement error for inflation due to the estimation of the shock in the standard Taylor Rule

The estimation of $\epsilon_{r,s}$ as depicted by the two series on the right side of the figure absorbs a huge amount of the drop of $\epsilon_{pi,ME}$. In the Taylor rule, the negative shift downwards of the disturbance amounts to an estimated policy rate below the one implied by the coefficients in the Taylor rule without the estimation. Likewise, the estimated measurement error on inflation does not need to fall as much in order to justify the loose stance of monetary policy observed in the data. The reason can be presumably encountered in the deviation of the actual, too accommodative monetary policy of the ECB in comparison to the monetary policy assumed by the standard Taylor Rule in the model. With reference to the BIS (2018), Taylor (2019) shows the deviation of the actual monetary policies from the mean of various Taylor Rules for the bulk of countries in the world. In other words, the assumed coefficients of ρ_s, ρ_y and ρ_π do not replicate the monetary policy of the ECB as derived from the time series on inflation and the policy rate, in particular, and the model dynamics affecting these variables. It remains for further research to not only estimate a small subset of parameters, i.e. the ones attached directly to QE and the market pricing of bonds, but a much greater number of parameters in the model with the priors given by the literature and calibration. Thereby, the potential issue of overidentification of parameters is targeted.

The prior vs. posterior distributions:

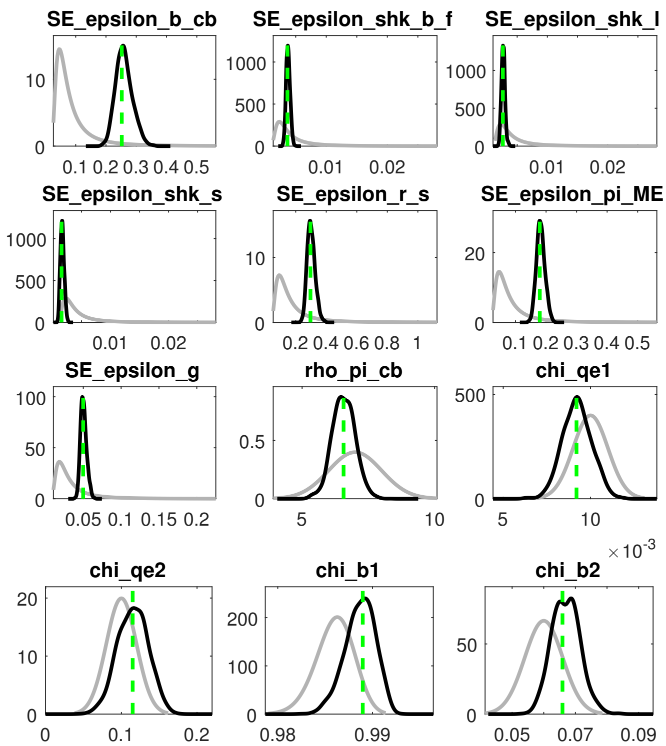

Figure 9: Priors and Posteriors.

Table 3: Results from posterior maximization (parameters)

	Prior distribution	Prior mean	Prior s.d.	Posterior mode	s.d.
$\rho_{\pi,cb}$	norm	7.000	1.0000	6.5650	0.4383
χ_{qe1}	norm	0.010	0.0010	0.0092	0.0010
χ_{qe2}	norm	0.100	0.0200	0.1149	0.0197
chi_{b1}	beta	0.986	0.0020	0.9890	0.0015
chi_{b2}	norm	0.060	0.0060	0.0659	0.0050

Table 4: Results from posterior maximization (standard deviation of structural shocks)

	Prior distribution	Prior mean	Prior s.d.	Posterior mode	s.d.
$\epsilon_{b,cb}$	invg	0.100	Inf	0.2522	0.0260
$\epsilon_{shk,b,f}$	invg	0.005	Inf	0.0036	0.0003
$\epsilon_{shk,l}$	invg	0.005	Inf	0.0026	0.0003
$\epsilon_{shk,s}$	invg	0.005	Inf	0.0016	0.0003
$\epsilon_{r,s}$	invg	0.200	Inf	0.2966	0.0237
$\epsilon_{\pi,ME}$	invg	0.100	Inf	0.1795	0.0141
ϵ_{g}	invg	0.040	Inf	0.0498	0.0039

Data Sources In sequence, the description and the source of data used for calibration and estimation is listed. Due to the estimation of a subset of parameters, some steady state values do not correspond exactly to the long run averages observed in the data. All steady state values are provided in the section 5.2. The data on government spending is only available from Q4 2002 onward, not from Q1 2000 onward.

Debt to GDP ratio in percent (Maastricht Criteria). Source: ECB Statistical Data Warehouse.
Government total expenditure in percent of GDP. Source: ECB Statistical Data Warehouse.

Tax-to-GDP ratio. Source: OECD.

Long-term interest rate for convergence purposes - 10 years maturity. Source: ECB Statistical Data Warehouse.

Bank interest rates - loans to corporations (new business). Source: ECB Statistical Data Warehouse.

Bank interest rates - overnight deposits from households. Source: ECB Statistical Data Warehouse.

Gross domestic product - expenditure approach. Source: Destatis cit. in OECD.

Private final consumption. Source: Destatis cit. in OECD.

Gross fixed capital formation. Source: Destatis cit. in OECD.

Credit (computation given in section 7). Source: BIS Statistics Explorer.

HICP Inflation. Source: BIS Statistics Explorer.

Public sector purchase programme (PSPP). Source: ECB Website, Questions and Answers.

Pandemic emergency purchase programme (PEPP). Source: ECB Website, Questions and Answers.

Appendix D: Bayesian IRFs

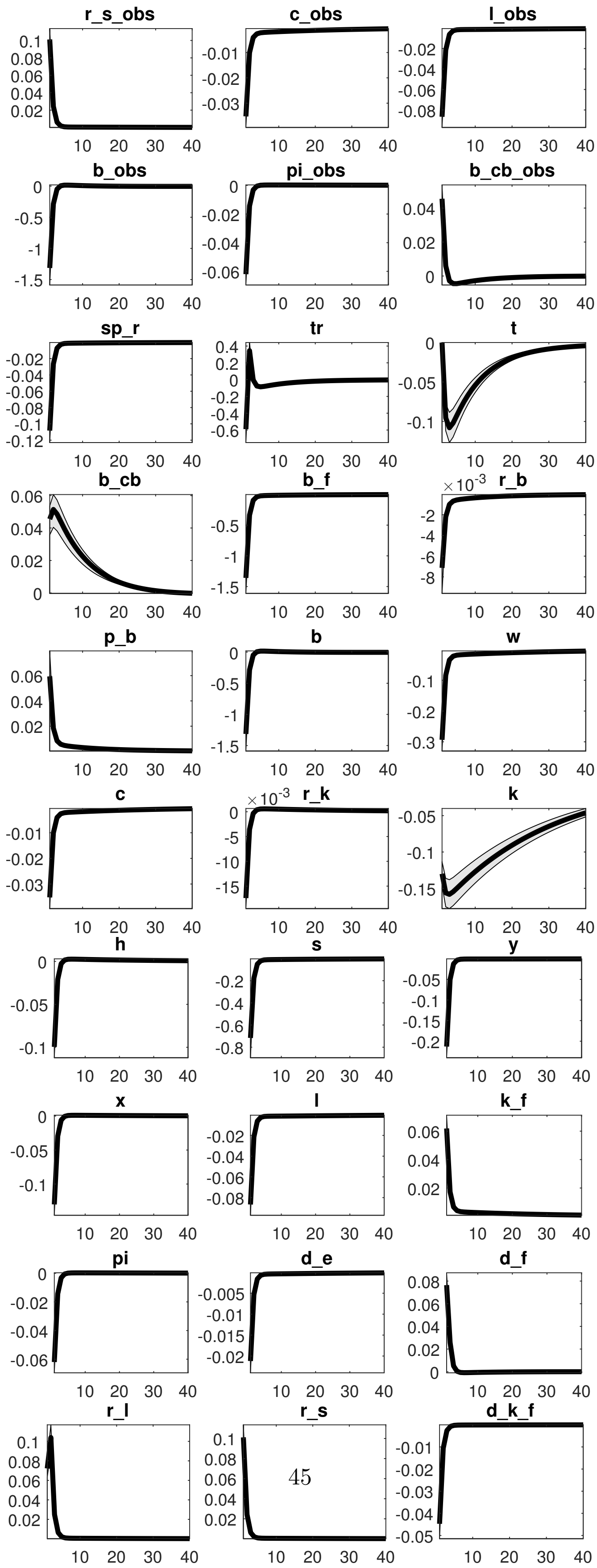

Figure 10: Bayesian IRFs Positive Monetary Shock.

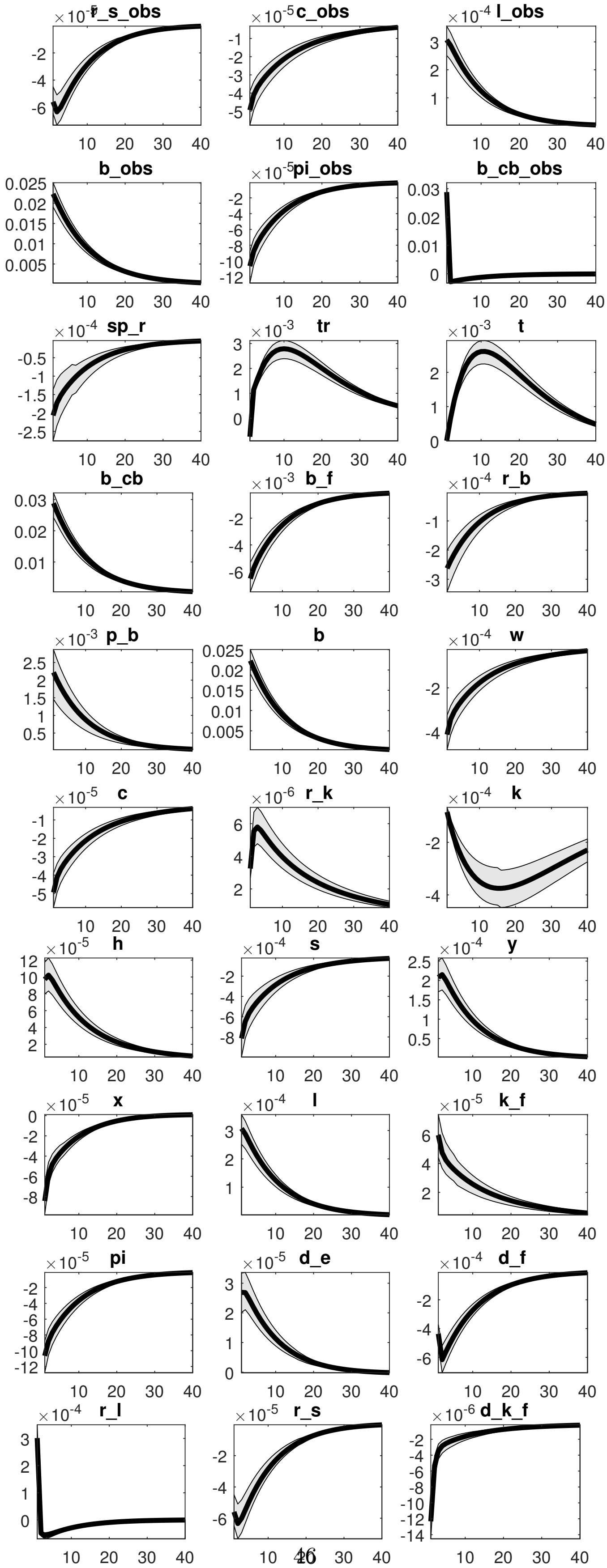

Figure 11: Bayesian IRFs QE Shock.

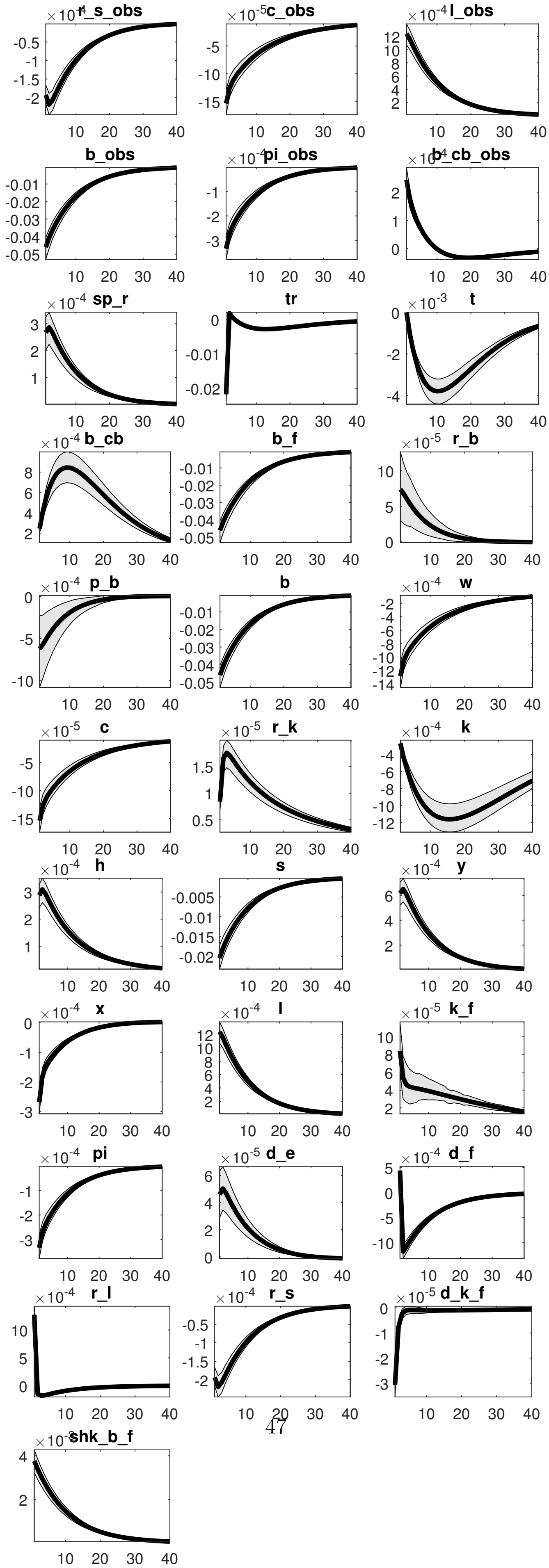

Figure 12: Bayesian IRFs Shock to Public Debt Exposure of Financial Intermediaries.

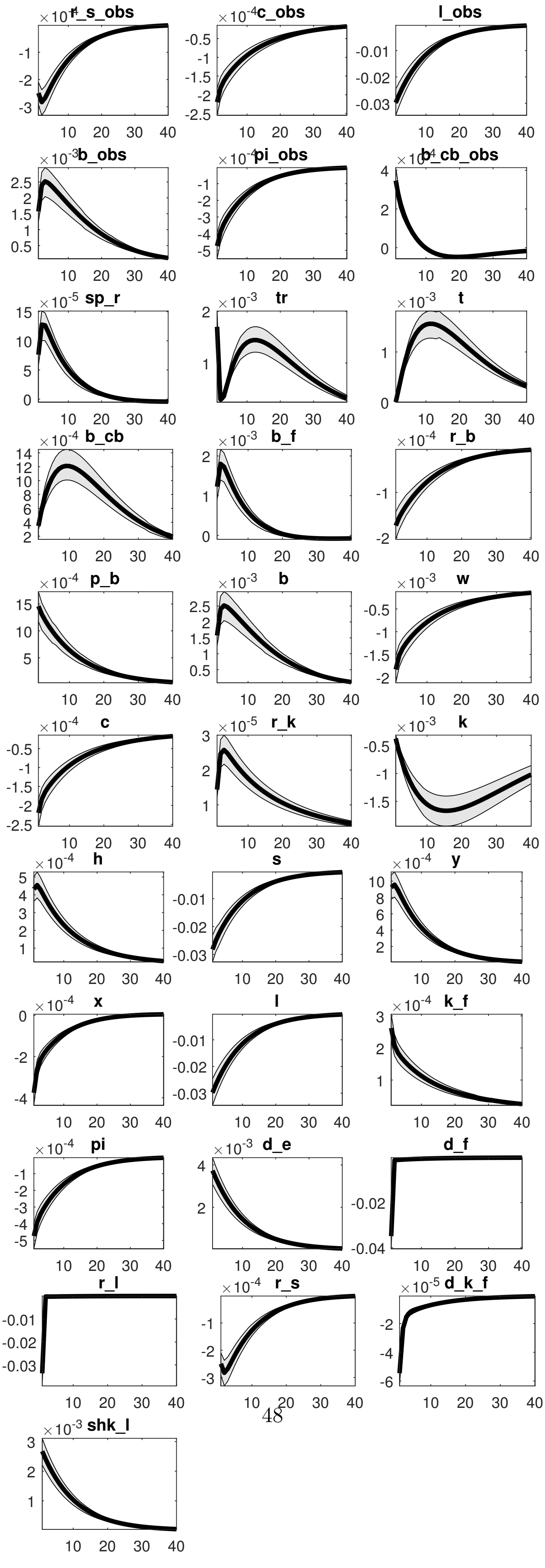

Figure 13: Bayesian IRFs Shock to Credit Supply.

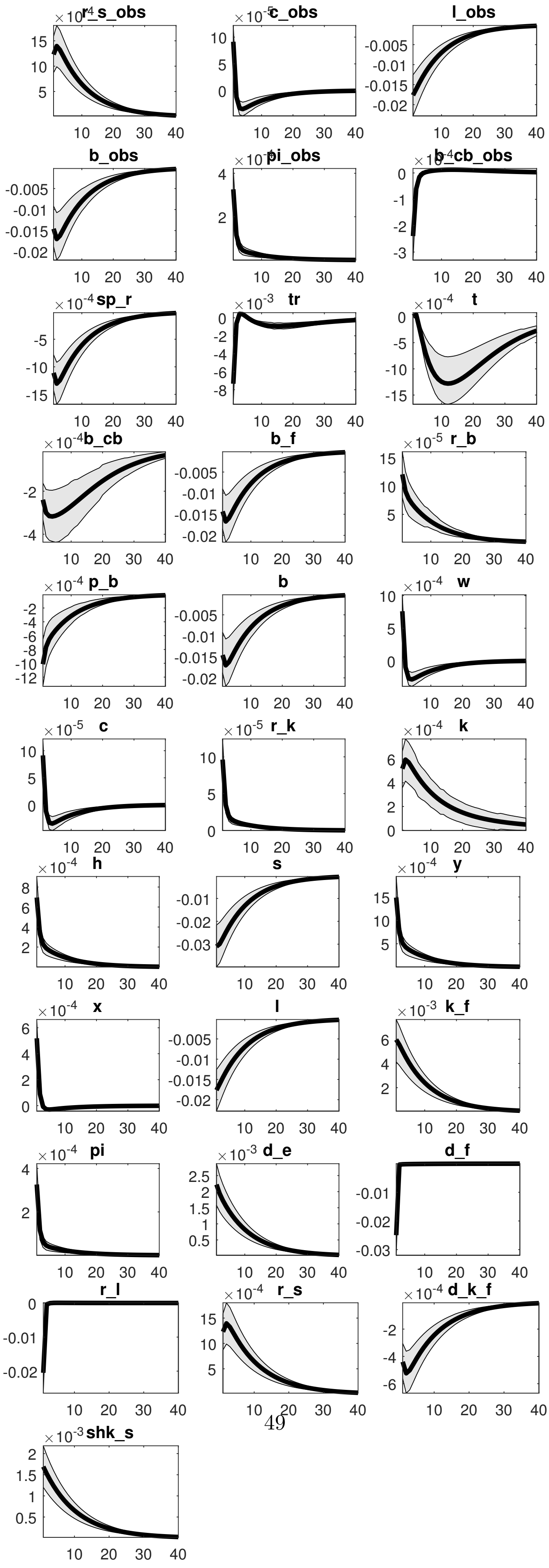

Figure 14: Bayesian IRFs Shock Bank Lending Channel.

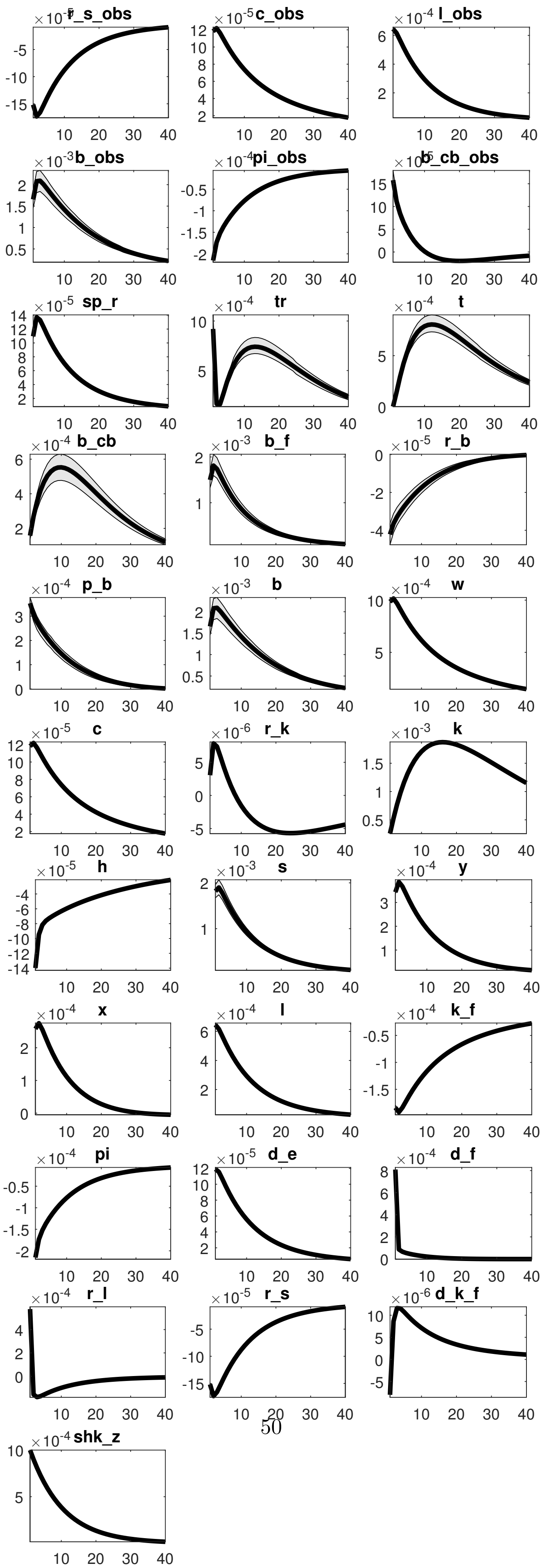

Figure 15: Bayesian IRFs Productivity Shock.

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